STEREOMETRY

Or, the Art of

GAUGING

Made easie by the Help of a

New Sliding-Rule:

WHICH SHEWS

The Area's of Circles in Gallons and Barrels and the Square and Cube-Root of any Number under 100000 by inspection; and also Resolves many other Arithmetical Problems without Pen or Compasses.

The Second Edition, carefully Corrected, and much Enlarged.

WITH AN

APPENDIX

OF

Conick Sections,

IN WHICH

The Nature and Original of several Solids (frequently mentioned in Books of Gauging) is Explained, and their Magnitudes compared.

By *THO. EVERARD*, Philomath.

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THE PREFACE TO THE READER.

IN the first Impression of this Tract, I signified what inducements I then had to make it publick: and the kind reception which that met with, hath incouraged me to publish this second Edition, for which I shall not at present make any Apology: for if this little Tract be in any measure serviceable to the end

for which it was designed, viz. *to instruct and assist his Majesties Officers for the Excise, (especially Supervisors and Gaugers) in the ready and faithful discharge of their Duties, to describe and distinguish the various forms of Casks, and to recommend to their Practice proper Rules for finding the true Content of each; that so neither his Majesty, nor the Merchant may receive that wrong in the Gauging of Brandy, and other Exciseable Liquors, which one of them must necessarily be supposed to do, if the false Rules, which some late Writers have given, be frequently made use of. I say if this Tract be truly serviceable to this end, it doth not (I conceive) need any Apology, and if it be not, I cannot make*

make any that will be ſufficient: and thus I might put an end to this Preface, but that I think it requiſite to give a ſhort account of the Additions I have made to this Impreſſion.

In the firſt Edition I inſerted ſeveral Problems, which (though uſeful in themſelves) were by ſome thought unneceſſary in a Tract of this Subject, theſe are omitted in this ſecond Edition, and ſeveral other things added, which I conceive are more uſeful, at leaſt they have a nearer relation to the Subject I Treat of. As for inſtance, in Sect. VIII. *I have laid down a Rule for Inching a Tun by Calculation.*

In

To the Reader.

In Sect. IX. I have endeavoured to describe and distinguish the various forms of Casks, and having noted the insufficiency of some of the common Rules, I have laid down others, by which the Content of any Cask may be readily found, either by the Pen or Instrument: here is also a Table of the Segments of a Circle, with its Construction, and Use in finding the Ullage of a Cask, with two other Tables very useful for the same purpose.

In this Section you have also a Rule for finding the Diameters of the Frustum of a Spheroid, and its Content upon every Inch. Lastly,

In an Appendix to this Edition, I have endeavoured to explain the Nature and Original of several Solids;

as

*us Spheroids, Parabolick and Hyperbolick Conoids, Parabolick and Hyperbolick Spindles, and given variety of Rules for finding the Content of each, either the whole or the parts: a more full and clear account of these and several other Solids may be seen in the Writings * of the Learned Dr. Wallis, Professor of Geometry in the University of* Oxford, *(to whom I must acknowledge my self very much obliged.) What is offered in this Appendix, and in the Book it self, will I hope answer the end for which I intend it: I have not vanity enough to think it will please all; but the Candid and Ingenuous (and*

* De Sectionibus Conicis. Mechanica, Pars Secund. Treatise of Algebra.

I va-

To the Reader.

I value not the censure of others) will (I doubt not) afford it a favourable reception.

Southampton
Octob. 9. 1688.

Farewel.

T. E.

THE

ADVERTISEMENT.

THE Inſtruments deſcrib'd in this Book are only made by *Iſaac Carver*, at the Sign of the *Globe-Dial* in *Horſly-down*: who alſo maketh all other Mathematical Inſtruments in Silver, Braſs, Ivory or Wood.

THE INTRODUCTION OF Decimal Arithmetick.

WHoever will rightly understand the Art of *Gauging*, or the *Use* of the *Instrument* here describ'd, ought in some measure to be acquainted with the Art of *Arithmetick*; at least with *Numeration*, *Addition*, *Subtraction*, *Multiplication* and *Division*, both in *whole Numbers*, and *Decimal Fractions*; the latter of which (if well understood) is in this Art most useful.

And forasmuch as most of the *Problems* in this Tract are resolv'd by help of the *Line of Numbers*, the several *Uses* of which *Line* are render'd more easie by the knowledge of *Decimal Arithmetick*: I shall in the first place endeavour to shew what a *Decimal Fraction* is, and also give some Rules and Examples, by which any person who does at all understand the vulgar *Arithmetick*, may in a few hours time thoroughly comprehend this.

A *Decimal Fraction*, is that, which by prefixing a point or a prick towards the left hand, its value is decreased from so many Units, to so many Tenth

parts of an Unit, and if a point and a Cypher, or a Digit be prefixed it will then be ſo many hundred parts, and if a point and two Cyphers or Digits be prefixed, its value is decreaſed to be but ſo many thouſand parts; as if you would prefix before the Figure 2, a point thus [.2] 'tis then decreaſed from 2 Units to 2 tenth parts of an Unit, and if you prefix a point and a Cypher thus, [.02] it is decreaſed from 2 Units to 2 hundred parts of an Unit. For,

As in *whole Numbers*, the value or denomination of places *increaſes* by Tens from the Units place toward the *left* hand, ſo in *Decimal Fractions*, the value or denomination of places *decreaſes* by Tens from the Units place towards the *right* hand, as in the following Scheme.

In which I have ſet an Unit in its due place, and all the Figures towards the left hand (being whole Numbers) do *increaſe* by Tens, for the Figure 2, next the Units place is 2 Tens, (or 20) the next is 3 hundreds, and the next 4 thouſands, *&c.*

Hundreds of Thouſands.	*Tens of Thouſands.*	*Thouſands.*	*Hundreds.*	*Tens.*	*Unit.*	*Tenth Parts.*	*Hundred Parts.*	*Thouſand Parts.*	*Ten Thouſand Parts.*	*Hundred Thouſ. Parts.*
6	5	4	3	2	1	.2	3	4	5	6

But all the Figures from the Units place towards the right hand (being Decimal Fractions) do *decreaſe* by

by Tens, ſo the Figure 2, towards the right hand, is 2 tenth parts, the next is 3 hundred parts, and the next 4 thouſand parts of an Unit, each place towards the right hand being ten times leſs.

In *Decimal Arithmetick*, we always imagin (and it would be very commodious if it were really ſo) that all intire Units, Integers, and things are divided into Ten equal parts, and each of theſe parts ſo divided we call *Primes*, we alſo divide each of theſe *Primes* into other Ten equal parts, and every of theſe diviſions we call *ſeconds*, and each of theſe being divided into Ten other equal parts may be called *thirds*, (and as one *Prime* is $\frac{1}{10}$ of an Unit, ſo one Second is $\frac{1}{10}$ of $\frac{1}{10}$ or $\frac{1}{100}$ part of an Unit, and one third is $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$ or $\frac{1}{1000}$ part of an Unit) and ſo by dividing the former, and ſubdividing theſe latter, we may run on *ad Infinitum*.

Let a Pound Sterling be given to be Decimally divided:

According to the notion premiſed, the firſt diviſion muſt be *primes*, the next diviſion *ſeconds*, the next *thirds*, &c.

So one Pound Sterling being 20 *s.* which divided into Ten equal parts, one of theſe parts is one *prime*, and its value is 2 *s.* and will ſtand thus, [.1] three *primes*, or three *tenths* of a Pound will ſtand thus, [.3] and its value is 6 *s.* Again one *prime* or .1, being divided into Ten equal parts, each of thoſe parts will be one *ſecond*, or one *hundred* part of a Pound, and is thus expreſſed, [.01] and its value will be 2 *d.* $\frac{1}{2}$ and $\frac{2}{10}$ of a Farthing; and if .01 be divided into Ten other equal parts, each of thoſe parts ſo divided will be *thirds*, or one *thouſand* part of a Pound, and will ſtand thus, [.001] and its value will be $\frac{24}{100}$ of a Farthing.

The like may be understood of one Pound *Troy* or *Averdupoize*, one Foot, one Gallon, or any other Integer or thing whatsoever.

A *Decimal* Fraction, whether it stand alone or be joyned with *whole* Numbers, is ever distinguished from a *whole* Number, by a point or a prick before it towards the left hand, as in these Examples.

.4752 4.752 47.52 475.2

In Decimal Fractions the *Numerators* only are set down, the *Denominators* being known by the *Number* of places in the *Numerator*, for if the *Numerator* consist of but one place, the *Denominator* is 10, if of 2 places the *Denominator* is 100, if of 3, 1000, if of 4, 10000, *&c.*

Example.

The *Denominator* of { .2, .25, .051, .0752 } is { 10, 100, 1000, 10000 }

As Cyphers *before* a whole number have no value, so Cyphers *after* a *Decimal* Fraction are of no signification, and therefore will not increase the Fraction nor alter the *Denominator*, for .2 is two tenth parts, and .20 is no more.

Again, as Cyphers *after* a whole Number do *increase* that Number, so Cyphers *before* a Decimal Fraction do *diminish* the value thereof, as in these *Examples.*

.5
.05
.005
.0005 } is 5 { *Tenth*
Hundred
Thouſand
Ten Thouſand } Parts.

Here every Cypher added does remove the Fraction further from Unity, making it Ten times leſs than before.

Addition and Subtraction of Decimal Fractions.

1. AS for the operation of *Addition* and *Subtraction* in *Decimals*, it is the *very ſame* with the *vulgar*, there muſt only care be had of placing *Units* under *Units*, and *Fraction* under *Fraction*, in their proper Ranks and Files.

For Example.

To ——— 376.43
Add ——— 54.20

Sum is ——— 430.63

Again,

To ——— 26.502
Add ——— 310.005

Sum is ——— 336.507

2. *Example in Subtraction.*

From	763.25
Subtract	296.31
Remains	466.94

Or, thus:

From	5.0525
Subtract	4.8508
Remains	0.2017

Note, In *Addition* there muſt ever be as *many* places of Fractions in the *Total*, as are found in any of the Sums to be added together.

Multiplication of Decimals.

1. IN *Multiplication* both of *Decimal* Fractions and *mixt* Numbers, there are ever *two* Numbers given to find a *third* unknown.

One of the Numbers given (and it's no matter which) is called the *Multiplicator*, the other the *Multiplicand*, and the Number ſought is called the *Product*, and this doth ever contain one of the Numbers given, as many times as the other given Number does contain Unity.

Multiplication of *Decimals* is perform'd after the *ſame manner* as in *whole Numbers*; for the Numbers being ſet down one under another, we proceed in the Multiplication as if they were all whole Numbers, only

only it muſt be obſerved (when the operation is finiſh'd) how *many* places of Decimal Fractions are contain'd in the *Multiplicator* and *Multiplicand*, for *ſo many* muſt be in the *Product*, as in theſe Examples.

Multiplicand —— 35.25
Multiplicator —— 7.24

14100
7050
24675

Product —— 255.2100

Or, Thus:

Multiplicand —— 6.75
Multiplicator —— .25

3375
1350

Product —— 1.6875

2. When it happens (as it often does) that there are not ſo many *Figures* in the *Product*, as there are *Fractions* in the two Numbers to be Multiplied, you are then to place *Cyphers* before the *Product*, 'till the *Number* of places be equal, as in theſe Examples.

.04	.75	8.5
.6	.05	.005
.024	.0375	.0425

3. As *whole Numbers* are increaſed by Multiplication, ſo *Decimal Fractions* are hereby made leſs, for the *Product* is removed farther from Unity than either of the Fractions given to be Multiplied, as appears by the laſt Examples.

4. When a *Decimal* Fraction or *mixt* Number is to be Multiplied by an Unit with Cyphers (as 10, 100, 1000, *&c.*) you need only remove the point ſo many places towards the right hand, as there are Cyphers with the Unit, thus if .7562 be Multiplied

By	10	the Product will be	7.562
	100		75.62
	1000		756.2
	10000		7562

Diviſion of Decimals.

1. IN *Diviſion* there are *two* Numbers given to find a *third* unknown.

One of the Numbers given is called the *Diviſor*, the other the *Dividend*, and the Number ſought is called the *Quotient*, and this *Quotient* doth ever contain Unity as many times as the *Diviſor* is contain'd in the *Dividend*.

Diviſion of *Decimal* Fractions or *mixt* Numbers, is perform'd after the *ſame manner* as in *whole* Numbers, and it being in either, more difficult than any of the former Species, I ſhall here inſert a few Examples together with ſuch directions as (I preſume) will render the Work both plain and eaſie.

Example.

Example.

Let it be required to divide 32 *by* 51.2

In this Example the *Diviſor* (51.2) is *greater* than (32) the *Dividend*, therefore in this and all the like caſes you muſt place a competent Number of *Cyphers* behind the *Dividend*, which (if it be a whole Number, the) Cyphers ſo placed muſt be diſtinguiſhed therefrom as Fractions, as here I put 4 Cyphers behind 32, and place the Numbers in this manner:

51.2) 32.0000 (

This done, I proceed to the diviſion (as if all were whole Numbers) and firſt ask how many times the *Diviſor* is contain'd in (320) the three firſt Figures of the *Dividend*, and ſeeing in this Example it is not contain'd in the three firſt Figures, it will therefore extend to the fourth place, over which (for diſtinction ſake) I put a prick, and then ask how many times 5 in 32: I find 6, I place 6 in the *Quotient*, and Multiply the *Diviſor* by 6, Subtracting the Product out of the *Dividend*, beginning my Subtraction at the place where I put the prick: Thus 6 times 2 is 12, 12 from 0 I cannot, I borrow 2 and ſay 12 from 20 and there remains 8, which I place under the Cypher: again, 6 times 1 is 6, and 2 I borrowed is 8; 8 from 10, and there reſts 2, which I place under the next Cypher before 8, as in the Example; then 6 times 5 is 30, and 1 I borrowed is 31; 31 from 32, and there reſts 1, which I ſet under 2; then for my new Dividend I bring down the next Cypher and ſet it be-

51.2) 32.0000 (6
128

hind the Remainder, and the Example will ſtand thus:

51.2) 32.0000 (6
1280

Then proceeding in my Diviſion, I ask how many times 5 in 12, I find 2, which I place in the *Quotient*, and ſay 2 times 2 is 4; 4 from 10, and there remains 6, which I ſet under 0: again, 2 times 1 is 2, and 1 I borrowed is 3; 3 from 8, and there reſts 5, which I place under 8, then 2 times 5 is 10; 10 from 12, and there reſts 2, which I place under 2, and to this Remainder I bring down another Cypher to make my new *Devidend*, ſo will the Example ſtand thus:

51.2) 32.0000 (62
1280
256

51.2) 32.0000 (62
1280
2560

This done, I inquire how many times 5 in 25, and find 5, which place in the *Quotient*, and ſay, 5 times 2 is 10, 10 from 10 and there reſts 0; then 5 times 1 is 5, and 1 I borrowed is 6, 6 from 6 and there reſts 0; laſtly 5 times 5 is 25, 25 from 25 and there remains 0; and thus my Diviſion is finiſh'd, and the Example will ſtand thus:

51.2) 32.0000 (.625
1280
2560
0000

Now

Now to know the Value or Denomination of places in the *Quotient* is the only difficulty, for reſolving of which there are ſeveral Rules; the moſt general is this following.

The firſt Figure in the Quotient is always of the ſame Denomination with that Figure which ſtands (or is ſuppoſed to ſtand) over the Units place in the Diviſor.

Therefore place the *Diviſor* (or ſuppoſe it to ſtand) under the *firſt* Figures of the *Dividend*, as it ought to ſtand if it were to be ſubtracted therefrom, in this Example it will ſtand thus:

32.0000 (.625
51.2

By which it appears, that the *Cypher* in the *Dividend* which ſtands over the *Units place* in the *Diviſor*, is the place of *tenths*; I therefore conclude, according to the Rule above given, that the firſt Figure in the *Quotient* muſt be of the *ſame* Denomination, (*viz.*) *tenths*; I therefore put a prick before it, ſo will the *Quotient* be .625:

And for the ſame reaſon, if the *Diviſor* were 512, the *Quotient* would be .0625, for in this caſe the *Cypher* which is *ſuppoſed* to ſtand over the *Units place* in the *Diviſor*, is the place of *hundredth parts*; therefore I put a Cypher before 6 in the *Quotient*, and then the Example will ſtand thus:

32.0000 (.0625
512

In like manner if the *Diviſor* were a Fraction, *viz.* .512, the *Units place* being next towards the left hand will ſtand under the place of *Tens* in the *Dividend*, and

and ſo the firſt Figure in the *Quotient* will be *Tens*, and ſtand thus:

32.0000 (62.5
.512

This may be further illuſtrated by the following Examples:

1. Example.

.0512) 032.0000 (625
1280
2560
0000

2. Example.

.04) 03.6 (90
00

.75) .0775 (.05
000

2. In Diviſion of *whole* Numbers or *mixt*, if there happen to be a Remainder, you may bring down more Cyphers, and by continuing your Diviſion, carry on the *Quotient* to as many places of Fractions as you pleaſe, four or five will be very near the truth, but two or three will be ſufficient in moſt Caſes, Example:

6.26) 25.800000 (4.1214
760
1340
880
2540
36

3. When any Number (either *Decimal* or *mixt*) is to be divided by an Unit with Cyphers as 10, 100, 1000,

1000, &c. you need only *remove* the prick in the *Dividend*, ſo *many* places towards the left hand as there are *Cyphers* with the Unit, ſupplying the *vacant* places (if any be) with Cyphers.

Thus 756.2 divided by 10, is 75.62, Divided by a 100 it will be 7.562, by 1000, .7562, and if by a 10000 the *Quotient* will be .07562.

4. *Diviſion* is *prov'd* by *Multiplication*; and *Multiplication* by *Diviſion*.

To *prove Diviſion*, Multiply the *Quotient* by the *Diviſor*, and the *Product* is the *Dividend*: Thus, .652 (the *Quotient* in the firſt Example of *Diviſion*) being Multiplied by 51.2 (the *Diviſor*) the Product is 32.0000 (the *Dividend*.)

To *prove Multiplication*; divide the *Product* by *either* of the Numbers given to be Multiplied, and the *Quotient* will be the other Number given.

Thus if the laſt Product (*viz.*) 32.0000 be Divided by 51.2 the *Quotient* is .625, or if 32.000 be Divided by .625, the *Quotient* will be 51.2.

Hence 'tis evident, that *Diviſion* is alſo *prov'd* by *Diviſion*: for if any *Quotient* be made a *Diviſor*, it will Quote the firſt *Diviſor*, Example:

```
1.8) 28.8 (16
     108
     000

16) 28.8 (1.8
     128
     000
```

5. Now for as much as *Multiplication* is *eaſier* than *Diviſion*; I ſhall here ſhew that what is perform'd by a *Diviſor*, may alſo be perform'd by a *Multiplicator*, and *how* ſuch a *Multiplicator* may be found:

found: Suppose I were to *Divide* 900 by 25, the *Quotient* will be 36.

Now let it be required to *Multiply* 900 by a certain Number that shall produce 36.

To *find* this *Multiplicator*, *Divide* an Unit with Cyphers by 25, the *Divisor* proposed, the *Quotient* will be .04 the *Multiplicator* sought; for 900 being *Multiplied* by .04 the *Product* will be 36, which was required.

Again, Having a *Multiplicator* to find a *Divisor*, this is but the *Converse* of the former, for if an Unit with Cyphers be *Divided* by .04 (a *Multiplicator*) the *Quotient* will be 25, the *Divisor* as before.

6. A *Vulgar* Fraction is reduced into a *Decimal* by *Division*: for which the Rule is.

Divide the *Numerator* (of the *Vulgar* Fraction given) by the *Denominator*, and the *Quotient* will be a *Decimal* Fraction, *equal* in Value to the *Vulgar* Fraction given: So $\frac{3}{4}$ reduced as aforesaid will be .75:

Example.

4) 3.00 (.75
20
00

Note, The *odd parts* of an Integer cannot be exactly reduced to a *Decimal*, for there will always be a *Remainder*, in such Cases carry on the Fraction to four or five places, it will be very near the truth:

Example.

Example.

$\frac{4}{9}$ 9) 4.0000 (4444
40
40
40

7. To reduce a *Decimal Fraction* to the *known parts* of the *Integer*:

If the *Decimal* given be part of a *Pound* Sterling, Multiply it by 20 (the *Shillings* in a Pound, and the Fractions in the Product by 12, the Pence in a Shilling) and the Fractions in this Product by 4, (the *Farthings* in a Peny.) This done, the *whole* Numbers in the respective Products shew the *Shillings*, *Pence*, and *Farthings* contained in the Decimal given, and the *Fractions* in the last Product are Decimals of a Farthing. *Example*: Suppose .25 a Decimal Fraction of a Pound Sterling were given to be reduced, Multiply it by 20, the Number of Shillings in a Pound, the Product will be 5, *viz.* 5*s.*

.25
20

Shillings ——— 5.00

So the *Value* of .60625 will be found to be 12*s.* 1 $\frac{1}{2}$ *d.* as appears by the following Example:

.60625
20

Shilllings ——— 12.12500

Shillings

Shillings —— 12.12500
12

25000
12500

Pence —— 1.50000
4

Farthings —— 2.00000

If the Decimal given be part of a *Beer Barrel*: Multiply it by 4, (the *Firkins* in a Barrel,) and the Fractions in the Product by 9, (the *Gallons* in a Firkin,) and the Fractions in that Product by 8, (the *Pints* in a Gallon.) This done, the *whole* Numbers in the respective Products are equal to *Firkins*, *Gallons* and *Pints* contained in the Decimal propounded: Example.

Let .72 be a Decimal of a Beer Barrel to be reduced as aforesaid, it will be found to be 2 Firkins 7 Gallons 7 Pints and .36 of a Pint, as appears by the Work.

.72
4

Firkings —— 2.88
9

Gallons —— 7.92
8

Pints —— 7.36

But

But *theſe* and *many other* Arithmetical Queſtions may be more *readily reſolved* by the *Inſtrument*, a deſcription of which ſee in the firſt Chapter of the following Diſcourſe.

ADVER-

ADVERTISEMENT.

IN the first Impression of this Tract, I gave the Explanation and Use of another Sliding-Rule of three Foot long, with one Sliding-Rod, that may be drawn out to six Foot; these with the Cane in which they are usually put, will extend to nine Foot in length, and are very useful in taking the Diameters of Tuns, and by reason of its length will resolve most Questions truer than the Foot Rule here described: but the Lines upon that being the same that are upon this, what is said of one may be understood of the other; this Foot Rule is sufficient for ordinary Practice. The Reader may use either, or both as he thinks fit.

STE-

STEREOMETRY

OR, THE

ART of GAUGING

MADE EASIE

By the Help of a New SLIDING-RULE.

SECT. I.

A Description of the Rule, and of the several Lines upon it, with their Use in some Questions in Arithmetick.

THE Rule consists of three Parts, *viz.* a Rule of 12 Inches long, and two small Scales to slide in it, which may be drawn out, one towards the right hand, and the other towards the left, till the whole be three Foot long.

The principal Lines on the Instrument are those commonly

commonly known by the name of *Gunters Line*, or *Line of Numbers*, which are here distinguish'd one from another by certain Letters, set at the end of the Lines towards the right hand.

Thus the Lines D are each of them one *single Line of Numbers*, beginning at the end of the Rule towards the left hand, and from thence continued to the other end.

The Lines A B and C are called *double Numbers*, each being *two* Lines or Radius's of Numbers, the *first* beginning at the *left* hand, and ending in the *middle* of the Rule, *where* the *second* Radius begins, and is from *thence* continued to the *end* at the *right* hand.

The Line E is called *Triple Numbers*, being *three* Radius's of Numbers; the *first* beginning at the *left* hand, and the *third* ending at the *right* hand.

This *Triple* Line is *equal* in length to the *double* Lines, and *all* to the *single* Line, for all the *five* begin and end at the *same* Point.

The *Line of Numbers* is (now) so *well known* to most persons, that it may be thought a *sufficient* description to have only said that *these* are *such*.

But for as much as this Instrument may be useful to some who (I presume) do not yet know what the *Line of Numbers* is, I shall therefore endeavour to explain it as followeth.

The *Line of Numbers*, is a Line of Geometrical proportions, divided first into *Nine unequal* parts call'd *Primes*, which are distinguish'd by Figures, 1, 2, 3, 4, 5, 6, 7, 8, 9; and then each of these *Primes* are subdivided into *ten* other parts (according to the same Reason) called *tenths*, and again, each of those *tenths* subdivided or at least supposed to be subdivided into *ten* other parts, according as the *length* of the Line will admit, as here the *Line* D, being about

11 In-

11 Inches long, each tenth in the first *Prime* is really subdivided into *ten* parts, call'd *Centesms*, but betwixt the Figures 2 and 4 each *tenth* is divided but into five parts, and therefore each of those parts do signifie *two Centesms*: again from the Figure 4 to the end of the Line every tenth is divided into two parts, each representing five *Centesms*: Lastly, each of these *Centesms* is also supposed to be divided into *ten* parts, which by some are call'd *Millians*; but a Line of this length will not admit of this last division.

The Figures (1, 2, 3, 4, 5, 6, &c.) by which the *Primes* are distinguish'd, are all Arbitrary points, and may each of them represent so many intire *Units*, *Tens*, *Hundreds*, or *Thousands*; or they may also represent so many *Tenth*, *Hundredth*, *Thousandth*, or *Ten Thousandth parts* of an *Unit*.

1. For *whole* or *intire Units*: Let the first *Prime*, or the Fig. 1. at the beginning of the Line, D represent one *Unit*, then shall all the Figures towards the right hand (*viz.* 2, 3, 4, 5, &c. to 10.) represent *so many* Units, and the tenths in each *Prime*, will be *tenth parts*, and the *Centesms* in each of those tenths will be *hundred parts* of an Unit.

Or, let 1 at the beginning of the Line represent 10 Units, then will each *Prime* forwards represent 10 times so many Units as the Figures express; thus, the Figures 2, 3, 4, 5, &c. will be 20, 30, 40, 50, &c.

And when one Prime represents 10 Units, each tenth in that *Prime* will be 1, and each *Centesm* in these tenths will be 1 tenth part of an Unit.

Again, Let the first *Prime* represent 100, then the Figures 2, 3, 4, 5, &c. will represent 200, 300, 400, 500, &c. and therefore 10 at the end will be 1000, and according to this supposition 1 tenth in each *Prime* will be 10 Units, and in those tenths each *Centesm* will be 1.

2. For

2. For *Decimal Fractions:* Let 10 at the end of the Line (at D) represent 1, then each *Prime* towards the left hand will be .1, and in those *Primes* each tenth will be .01, and in these tenths each *Centesm* will be .001 part of an Unit.

To make this more plain, draw out the sliding Piece B, till 1 at the *beginning* of the Line B, stand exactly against 10 at the *end* of the Line A, for then you have a Line of Numbers four times repeated; upon which let 1 at the *beginning* of the Line A, represent 1 Unit; then shall 1 in the *middle* of the said Line be 10, and 10 at the *end* thereof (or which is all one at the *beginning* of the Line B,) represent 100, and by consequence 1 in the *middle* of the Line B will be 1000, and for the *same* reason 10, at the *end* of the said Line (which is also the *end* of the *Rod*) will be 10000.

But keeping the Rule as it now stands, let 10 at the end of the fourth Radius (*viz.* at B) represent 1, then shall each Prime in the said fourth Radius represent .1, in the third Radius .01, in the second Radius .001, and in the first .0001 part of an Unit.

So will 2 in the first Radius be .0002, in the second 002, in the third .02, and in the fourth .2 parts.

The *Numbers* and *Divisions* on the Lines being thus explained, it will not (I presume be difficult to find the point upon the Line, where any Number given is represented.

As for Example, Suppose the Number were 1895. for the first Figure thereof (*viz.* 1.) I count 1 at the beginning of the Line D, for the second Figure I count 8 tenths next following (that is 8 of the greater Divisions betwixt 1 and 2, then from this point forward I count 9 *Centesms* for the third Figure, and for the last Figure 5 I count half the next *Centesm*; so I find the Point *a g* will represent 1895, and by the

the ſame Rule the Number 1715 will be found at the Point *w g*; hence obſerve,

1. That (on a Line of this length) only the *four firſt* Figures of any Number propoſed can be diſcovered, for if the Number given were 189562, it would be repreſented at the ſame Point where the former 1895 was found (*viz.*) at *a g*.

2. That all Numbers which after the firſt Figure have nothing but *Cyphers* (as 20, 200, 2000, *&c.*) are all repreſented at the *ſame* Point.

So 20, 200, 2000, are all repreſented by the Figure 2, at the beginning of the ſecond *Prime*.

3. All Numbers conſiſting of *three* Figures, and having a Cypher in the *middle*, are found within the firſt tenth of that *Prime*, at which the firſt Figure of the Number given is found.

Example, Let the Number given be 308, for the *firſt* Figure I count 3 on the Line, (which I find at the beginning of the third *Prime*) now there being a Cypher in the *ſecond* place, I muſt not count any of the tenths, but for the *laſt* Figure 8, I count 8 *Centeſms*, and that is the Point which doth repreſent 308.

4. All Numbers conſiſting of *four* places, and having *two* Cyphers in the *middle*, muſt be ſought betwixt the beginning of the *Prime*, unto which they belong, and the firſt *Centeſm* of the ſame *Prime*; ſo 4005 being given, the *firſt* Figure (*viz.*) 4, is found at the beginning of the fourth *Prime*: Now there being Cyphers in the ſecond and third places, I muſt not count any of the tenths or Centeſms, but for the *laſt* Figure 5, I eſtimate 5 Milliams, which is about the middle of the firſt *Centeſm*, and that is the Point where 4005 is repreſented.

Note, *Decimal* Fractions and *mixt* Numbers, are diſcovered after the *ſame* manner as whole Numbers,

bers, for if the first Number 1895, were 18, 95, it will be found at the *same point* (and by the *same rule* as the whole Number, (*viz.*) at *a g*, and by what hath been said, it is easie to find what Number is represented at any Point upon the Line, as will appear in the following Questions of Proportion, of which I come now to treat.

In *Arithmetick* (saith Mr. *Wingate*,) there are *three* several sorts of *Proportion*, *Arithmetical*, *Geometrical* and *Musical*.

Arithmetical, when *divers* Numbers being *compared* together retain amongst themselves *equal* differences, as these, 2, 4, 6, 8. *&c.* And this is either *continued*, as in the Numbers before propounded, or in these, 3, 6, 9, 12, *&c.* which is also called *Arithmetical progression*, or a *rank* of Numbers *Arithmetically* proportional; or *discontinued*, as in these 2, 4, 8, 10, or the like.

Geometrical proportion, is when *divers* Numbers being *compared* together *differ* amongst themselves, according to the same rate or reason, as these 2, 4, 8, 16, *&c.* for here, as 2 is half 4, so 4 is half 8, and 8 half 16; this is likewise either *continued*, as in those before propounded, or in these 1, 3, 9, 27, 81, *&c.* which are also call'd *Geometrical progression*, or a *rank* of Numbers *Geometrically* proportional; or *discontinued*, as in these 2, 4, 16, 32, for as 4 is double 2, so is 32 double 16, but so is not 16 being compared with 4.

Musical proportion, being of no use in the present business, I shall not trouble the Reader with it: But if the two *former* (viz. *Arithmetical* and *Geometrical*) be *duly* consider'd, the following Problems may be the *better* understood.

Probl. I.

Problem I.

Having two Numbers given, to find a third Geometrically proportional unto them, and to three a fourth, and to four a fifth, &c.

FInd *one* of the Numbers given upon the Line B, and ſet it againſt the *other* given Number on the Line A, then find the *ſame* Number upon B, (which was laſt counted upon A) and againſt it you have this *third* proportional upon A, and againſt this *third* upon B, is the *fourth* upon A; in like manner againſt the *fourth* upon B, you have the *fifth* upon A, *&c.*

Example, Let it be required to find a *third* proportional to *theſe two* Numbers 2 and 4, which may bear the *ſame* proportion to 4, that 4 bears to 2.

Draw out the ſliding Rod till 2 upon B ſtand againſt 4 upon A, this done againſt 4 upon B is 8, the *third* proportional, upon A, and againſt this *third* (*viz.* 8) upon B, is 16 upon A, which is the *fourth* proportional: Likewiſe againſt 16 upon B, is 32 the *fifth*, upon A, and againſt 32 upon B is 64 the *ſixth* proportional; but now proceeding forward, I find that 64 upon B, will reach *beyond* the end of the Line A, I therefore ſeek 64 towards the *left* hand upon B, and againſt it I find 128 the *ſeventh* proportional; and ſo proceeding farther you may find the *eighth* to be 256, the *ninth* 512, *&c. Contrariwiſe*, if it were required to find a *third* proportional to the *ſame* Numbers 2 and 4, which may bear the *ſame* proportion to 2, that 2 bears to 4.

Set 4 in the ſecond Radius upon A, to 2 upon B, then againſt 2 upon A (towards the left hand) is the

third proportional, and againſt 1 upon A is .5 the *fourth* upon B, alſo againſt this *fourth* (*viz.* 5) upon A is .25, the *fifth* proportional on B, *&c.*

In like manner, if the two Numbers given were 2 and 5; ſet 2 in the firſt Radius upon B, to 5 upon A, then againſt 5 upon B is 12.5, the *third* Proportional, and againſt this *third* upon B is 31.25, the *fourth* upon A, *&c.* But if you would find a *third* Proportional to the ſaid Numbers 2 and 5, which may bear the ſame Proportion to 2, that 2 bears to 5.

Set 5 in the ſecond Radius upon A, to 2 upon B, then againſt 2 (towards the left hand) upon A is .8, the *third* Proportional ſought; alſo againſt this *third* upon A, is the *fourth* upon B, *viz.* .32, and againſt .32 upon A, is .128 the *fifth* Proportional, *&c.*

Multiplication by the Lines.

Problem II.

One Number being given to be Multiplied by another, to find the Product.

IN Multiplication either of whole Numbers, mixt, or Decimal Fractions; the Proportion is:

As 1, *is to the Multiplicator;*
So is the Multiplicand, to the Product.

And the *Product* of any *two* Numbers ſhall have ſo *many* places as there be in *both* the Numbers given, except when the *leſſer* of them do *not* exceed ſo *many* of the *firſt* Figures of the Product; then it will have *one* leſs.

1. Ex-

1. Example: *Let it be required to Multiply* 6 *by* 4.

Say, [1 . 4 :: 6 . 24] which Analogy or Proportion may be read thus, *As* 1 is to 4, *So* is 6 to 24.

Therefore, Set 1 upon the Line B, to 4 on the Line A, and then againſt 6 upon B, is 24 upon A, which is the Product ſought.

Note, the *Unit* or *firſt* term may be taken upon *either* of the Lines A or B, but the *firſt* and *third* terms muſt be counted upon *one* and the *ſame* Line, and the *ſecond* on the *other* Line, where the *fourth* will alſo be found.

The Letters A and B, may ſerve to diſtinguiſh the Lines.

2. Example: *Let the two Numbers given be* 68 *and* 26, *to find the Product.* The proportion is, [1 . 26 :: 68 . 1768.]

Therefore,

Set 1 upon B to 26 upon A, then againſt 68 upon B, is 1768 on A, which is the Product ſought; Or.

Set 1 upon B, to 68 upon A, then againſt 26 upon B, is 1768 (on A) the Product. Therefore it matters not *which* of the Numbers given be made the *Multiplicator*; and *note* alſo, that the Product hath as many places as are in both the Numbers given, becauſe the leaſt of them (*viz.* 26) doth *exceed* ſo many of the firſt Figures of the Product, according to the Rule before given.

3. Example: *Let* 68 *be Multiplied by* 14. The proportion is,

1 . 14 :: 68 . 952

Therefore,

Set 1 upon B to 14 upon A; then against 68 upon B is 952 the Product upon A, and here the Product consists of one place *less* than there be in the two Numbers given, because the lesser of them (*viz.* 14) doth *not* exceed so many of the first Figures of the Product.

Now that the Product last found is 952, and not 95.2 nor 9.52 will thus appear: Set 1 upon B against 14 upon A,

And then against	2	upon	28	upon
	3		42	
	4	A is	56	B.
	5		70	
	6		84	
	6.8		95.2	

By this 'tis evident, that the Lines are in effect a Table of Multiplication, for having set 1 to the Multiplicator, against any Multiplicand you have the Product; so if 2 be Multiplied by 14, the Product is 28; if 3, the Product will be 42; if 4, 56; if 5, 70, *&c.* Hence I conclude, that if the Multiplicand had been but 6.8, the Product would have been 95.2, but the Multiplicand being 68, the Product must be 952. for by taking away the prick, the Fractions in each are made whole Numbers.

When of *two* Numbers given to be Multiplied, the *one* consists of *whole* Numbers or *mixt*, and the *other* of *Fractions* only, make the *whole* or *mixt* Number the *Multiplicator*, and having set 1 against it, seek the Fraction towards the left hand, for against it you have the *Product*.

4. Ex-

4. Example: *Let the two Numbers be* 27.5 *and* .8.

Set 1 on A, to 27.5 on B, and then againſt .8 (which being *leſs* than 1 I ſeek towards the left hand) on A is 22 the Product on B.

And notwithſtanding a Number of *more* than four places cannot be exactly expreſſed on a Line of this length, yet the Product of any Multiplication may be diſcovered to *ſix* or *ſeven* places at leaſt.

5. Example: *Suppoſe I were to Multiply* 2482 *by* 54. The proportion is,

1 . 54 :: 2482 . 134028

Therefore,

Set 1 upon A to 54 upon B, and then againſt 2 upon A, is 108 upon B: now ſuppoſe

2 be { 10 / 200 / 2000 } 108 will be { 1080 / 10800 / 108000 }

So the Product will have ſix places, and againſt 2482 the Multiplicand, you may diſcern (upon the Line) the four firſt of them, (*viz.*) 1340, the two laſt may be found by Multiplying (in ones mind) the two laſt Figures of the *Multiplicand*, by the two laſt of the *Multiplicator*; for ſo ſhall you diſcover the two laſt Figures of the *Product*, which in this Example will be 28, which placed behind the four firſt (*viz.*) 1340, makes the Product complete 134028.

I might here add more Examples, but theſe already given may ſerve, they containing (I hope) ſufficient directions for all the variety that can happen in Multiplication.

Division by the Lines.

Problem III.

One Number being given to be divided by another, to find the Quotient.

IN Division both of whole Numbers and mixt, the proportion is,

As the Divisor is to 1,
So is the Dividend to the Quotient.

Which Quotient shall ever consist of (but) so many Figures as the Dividend hath more than the Divisor, except when the Divisor does not exceed so many of the first Figures of the Dividend; then it shall have one place more.

1. Example: *Let it be required to divide* 24 *by* 4: The proportion is,

4 . 1 :: 24 . 6

Therefore,

Set 4 upon B to 1 upon A, and then against 24 upon B, is 6 upon A, which is the Quotient sought.

2. Example: *Let it be required to divide* 1768 *by* 26: The proportion is,

26 . 1 :: 1768 . 68

Therefore,

Set 26 upon B, to 1 upon A, and then against 1768 upon B, is 68 (the Quotient) upon A.

3. Ex-

3. Example: *Suppose* 952 *were to be divided by* 14: The proportion is,

14 . 1 :: 952 . 68

Therefore,

Set 14 upon A, to 1 upon B, then againſt 952 upon A, you have 68 upon B, which is the Quotient required.

Obſerve here, the Dividend hath but one Figure more than the Diviſor, yet the Quotient doth conſiſt of two Figures, becauſe the Diviſor does not exceed ſo many of the firſt Figures of the Dividend, but in the firſt and ſecond Examples the Quotients have but ſo many Figures as the Dividend hath more than the Diviſor, becauſe the Diviſor doth exceed ſo many of the firſt Figures of the Dividend, according to the general Rule above given.

This ſhews (in all Caſes) how many Figures muſt be in the Quotient, the value of the firſt of which may be found by the Rule given in the Introduction, page 11.

Or it will thus appear, that the Quotient in the laſt Example is 68, and not 6. 8, nor .68: Set 14 upon A, to 1 upon B,

And then againſt	28 42 56 70 84 95.2	upon A is	2 3 4 5 6 6.8	upon B.

This is but the Converſe of the third Example of Multiplication, for as by that it is manifeſt that if 2 were Multiplied by 14, the Product would be 28;

if 3, it would be 42; if 4, 56, &c. So here it is evident that if 28 be divided by 14, the Quotient will be 2; if 42, it will be 3; if 56, it is 4, &c. thus by reading on the proportion from the Divisor 14: 28, 56, 70, &c. I find at last, that if the Dividend were but 95. 2, the Quotient would be 6. 8, but the Dividend being 952, the Quotient must be 68, for by taking away the prick, the Fractions in each are made whole Numbers.

By these Examples it is also evident, that at *once* setting of the Rule we both *Multiply* and *Divide*.

For if 14 be a Multiplicator, set 1 on B, against 14 on A; this done, against any Multiplicand upon B, you have the Product upon A, as appear'd by the third Example of Multiplication.

And without moving the Rule, if you suppose 14 to be a Divisor, then against any Dividend upon A, you have the Quotient upon B, as in the last Example of Division.

How by *any Divisor* to find a *Multiplicator*, was shewn in page 14 of the *Introduction*; It may also be performed by the *Lines* more readily.

Thus *having* a Divisor to *find* a Multiplicator:

Set the Divisor given upon A, to 1 upon B, and then against 1 (towards the left hand) upon A, is the Multiplicator upon B. Example, suppose 25 were a *Divisor* given, to find a *Multiplicator*.

Set 25 upon A, to 1 upon B, and then against 1 (towards the left hand) upon A, is .04 the *Multiplicator* sought.

By a *Multiplicator* to find a *Divisor*: This is but the *Converse* of the former, for having .04 a Multiplicator upon B, set it against 1 upon A, and then against 1 upon B, is 25 the Divisor as before.

Problem

Problem IV.

To reduce a vulgar Fraction into a Decimal.

SET the Denominator (of the vulgar Fraction given) upon A, to the Numerator thereof upon B, and then againſt 1 towards the left hand upon B, is the Decimal Fraction ſought.

So $\frac{1}{4}$ will be found equal to this Decimal (*viz.*) .25 : For,

As 4 upon A is to 1 upon B; *So* is 1 upon A to .25 upon B:

Alſo $\frac{21}{28}$ will be .75, for *As* 28 upon A, is to 21 upon B; *So* is 1 upon A, to .75 upon B.

In like manner this Vulgar Fraction (*viz.*) $\frac{582}{960}$ is equal to this Decimal, *viz.* .60625, For, Set 960 upon A, to 582 upon B, then againſt 1 upon A, is .60625 upon B, and ſo for any other.

Problem V.

A Decimal Fraction being given, to reduce the ſame into the known parts of the Integer.

1. IF the Decimal be part of a Pound Sterling: Set 1 upon B, to the Number of Shillings, Pence or Farthings contained in a Pound upon A, then ſeek the Decimal given upon B, (towards the left hand) and againſt it you have the Shillings, Pence or Farthings (reſpectively) contain'd in the Decimal given.

Example: Suppose .7625 were to be reduced into Shillings.

Set 1 upon B to 20 (the Number of Shillings in a Pound) upon A, then against .7625 upon B, (towards the left hand) is 15. 25, that is, 15 *s.* and .25 parts of a Shilling, which is equal to 3 *d.*

2. To reduce the Decimal given into Pence: Set 1 upon B, to 240 (the Pence in a Pound) upon A, then against .7625 towards the left hand upon B, is 183, the Pence contain'd in .7625.

3. But if the Decimal aforesaid were to be reduced into Farthings.

Set 1 upon A, to 960 (the Farthings in a Pound) upon B, and then against .7625 upon A, is 732, and so many Farthings are contain'd in .7625.

Again, suppose .7625 were a Decimal of an Ale Barrel to be reduced into Gallons and Pints.

Set 1 upon A, to 32 (the Gallons in a Barrel) upon B, then against .7625 upon A, is 24.4, that is, 24 Gallons and .4 tenth parts of a Gallon; to reduce this .4 into Pints, set 1 upon A, to 8 upon B, and then against .4 upon A is 3.2 Pints: So .7625 of an Ale Barrel is equal to 24 Gallons 3 Pints and 2 tenth parts of a Pint.

Problem VI.

Three Numbers being given, to find a fourth in a direct proportion.

THis is call'd the *Rule of Three Direct*, and by the Instrument is wrought thus:

Set the first Number given upon B, to the second upon A, and then against the third Number given upon B, is the fourth Number sought.

Example:

Example: *If* 8 *Quarters of Mault will make* 20 *Barrels of Strong Beer, How many Barrels of ſuch Beer will* 22 *Quarters make?*

Set 8 upon B, to 20 upon A, and then againſt 22 upon B, is 55 upon A, and ſo many Barrels will 22 Quarters make, and according to this proportion 24 will make 60 Barrels, 28 will make 70, alſo 32 Quarts will make 80 Barrels, *&c.*

Problem VII.

To three Numbers given, to find a fourth in an inverſed proportion.

THis is called the *Rule of Three Inverſe*, in which obſerve, that if the *third* Number be *greater* than the *firſt*, then will the *fourth* be *leſs* than the *ſecond*. And contrariwiſe, if the *third* Number be *leſs* than the *firſt*, the *fourth* will be *greater* than the *ſecond*, and in either Caſes the Rule is this:

Set the third Number upon A, to the firſt (being of like denomination) upon B, and then againſt the ſecond Number upon A, you have the fourth upon B.

Example: *If* 8 *Men do any piece of Work in* 9 *days, In how many days can* 12 *Men do the ſame Work?*

Set 12 upon A, to 8 upon B, then againſt 9 upon A, is 6 upon B, which is the anſwer. For 12 Men may do the ſame Work in 6 days, that 8 Men do in 9 days.

But

But if the Question had been, *In how many days can 6 Men do the same Work?* the Answer will be 12: For,

As 6 upon B, is to 8 upon A, *So* is 9 upon B, to 12 upon A.

Problem VIII.

Betwixt two Numbers given, to find a mean Geometrically Proportional.

SET one of the Numbers given upon C, to the same Number upon D, and then against the other given Number upon C, is the Geometrical mean sought.

Example: *Let the Numbers given be* 50 *and* 72, *to find a Geometrical mean*, &c.

Set 50 upon C, to 50 upon D, and then against 72 upon C, is 60 upon D. So 60 is a Geometrical mean betwixt 50 and 72.

Or thus,

Set 72 upon C, to 72 upon D, and then against 50 upon C, is 60 upon D, the mean as before.

By the Pen thus:

Multiply one Number by the other, then Extract the Square-Root of the Product, this Square-Root is the Geometrical mean between the two Numbers given, thus 72 Multiplied by 50 is 3600, whose Square-Root is 60, the Geometrical mean between 50 and 72.

Problem

Problem IX.

To find the Square-Root of any Number under 1000000.

THE *Extraction of Roots* is one of the *hardeſt* Leſſons in *Arithmetick*, yet by help of this Inſtrument it may be perform'd with *leſs trouble* than any of the foregoing Problems: For if the Lines C and D, be applied one to another, ſo as 10 at the end of D, be even with 10 at the end of C; I ſay the Lines thus applied are like a Table ſhewing the Square-Root of any Number by Inſpection only; for againſt any Number upon C, you have the Square-Root thereof upon D, *Et Cout.*

Note, 1. When the Figures in the Number given are even, *viz.* when the Number conſiſts of 2, 4, 6 or 8 Figures (being Integers) look the ſame in the ſecond Radius of the Line C, and againſt it you have the Square-Root upon D: And in this caſe the ſaid Root will ever conſiſt of half as many Figures as the Number given.

Example: *Let* 16 *be the Number propounded*, I ſeek 16 in the ſecond Radius upon C, and againſt it upon D, I find 4 the Square-Root required.

In like manner { 5.5 / 48 / 886 } is the Root of { 30.25 / 2304 / 784996 }

2. When the Integers in the Number given are odd, *viz.* 1, 3, 5 or 7, ſeek it in the firſt Radius upon the Line C, and againſt it you have the Root ſought. And in this caſe the Root will have

have half as many Figures as the Numbers given and one more.

Example: *Let the Number given be* 156.25, I ſeek this upon the firſt Radius of the Line C, and againſt it I find 12.5 the Root ſought.

So alſo { 24, 144, 1000 } be the { 576, 20736, 1000000 } will Root of

Problem X.

To find the Cube-Root of any Number under 1000000000.

PLace the Lines D and E one by another, ſo as 10 at the end of D, be even with 10 at the end of E, this done againſt any Number upon E, you have the Cube-Root thereof upon D, *Et Cont.*

Note, 1. When the Number given conſiſts of 1, 4 or 7 Figures (being Integers) find it in the firſt Radius of the Line E, and againſt it you have the Cube-Root ſought. Example: Let the Number given be 3375, I ſeek this in the firſt Radius in the Line E, and againſt it I find 15 upon D, which is the Cube-Root of 3375, and ſo is 212 the Cube-Root of 9528128.

2. When the Number given conſiſts of 2, 5 or 8 Integers, find it in the ſecond Radius upon E, and againſt it is the Root ſought. Example: Suppoſe 35.937 were propounded, find this in the ſecond Radius of E, and againſt it is 3.3 the Cube-Root upon D; in like manner is 275 the Cube-Root of 20796875.

3. When

3. When the Number given consists of 3, 6 or 9 Integers, it must be sought in the third Radius, &c. For against it is the Cube-Root: Thus against 125 in the third Radius upon E, I find 5 the Cube-Root, and so likewise is 888 the Cube-Root 700227072.

Lastly, to know how many places of Integers must be in the Cube-Root of any Number given.

Put a point over the place of Units in the Number given, then omitting 2, point every third Figure toward the left hand, then tell how many points, for so many places of Integers must the Cube-Root consist of.

Sect. II.

Of a Superficies.

1. A *Superficies* is a Figure incompassed about with a Line or Lines, and is either Round or Angular.

2. A *Round Figure* is that which is contain'd by one Round Line, and is either a Circle (as Fig. 1.) or an Ellipsis (as Fig. 2.)

3. An *Angular Figure* is that which doth consist of three or more Angles, from the Number of which they are denominated, as a Figure of three Angles is called a Triangle, of four a Quadrangle, &c.

4. A *Triangle* is a Superficies comprehended by three Right-Lines (as Fig. 3.) of these there are six varieties, which I shall forbear to mention, they being all measured by one and the same Rule.

5. A

5. A *Quadrangle* is a Figure comprehended by four Right Lines, and is either a *Parallelogram* or *Trapezium*.

6. A *Parallelogram* is a Figure whose opposite sides are parallel, having equal distances from one another in all places, and is either Right or Oblique.

7. A *Right-Angled Parallelogram*, is that whose Angles are all right, and is either a Square (as Fig. 5.) or an Oblong (as Fig. 4.)

8. The *Oblique-Angled Parallelogram*, is that whose Angles are all Oblique, and is either a Rhombus (as Fig. 7.) or a Rhomboides (as Fig. 6.)

9. a *Trapezium*, is a Quadrangular Figure, whose four sides are not all equal, (as Fig. 8.)

10. *Figures* consisting of *more* sides than *four* are almost *innumerable*, but are reducible unto *two* sorts, *Regular* or *Irregular*, either of which are also call'd *Polygons*.

11. *Regular Polygons* are such whose sides and Angles are equal, they take their names from the Number of their sides, as that of five sides is call'd a *Pentagon*, (as Fig. 9.) that of six sides an *Hexagon*, &c. Of *Irregular Polygons*, 'tis needless to say any thing, they being measur'd after the same manner as a *Trapezium*.

I should next proceed to shew the Use of the Rule in the *measuring* of a Superficies, but it will not be altogether impertinent if I first premise;

That every *Magnitude* must be measur'd by some *known* kind of Magnitude that is *Homogeneal* (or like) to it. A *Line* is measur'd by a *Line*, as one Lineal Inch, Foot or Yard.

A *Superficies* is measur'd by a *Superficies*, as one Square Inch or Foot, *&c.* A *Solid* is measur'd by a *Solid*, as one Cubick Inch, one Cubick Foot, *&c.*

And

And when it is known how many	Lineal Inches or Feet,	are contain'd in a	Line.
	Square Inches or Feet,		Superficies.
	Cubical Inches or Feet,		Solid.

Then is the Quantity or Content of either of these kind of Magnitudes said to be known.

And the measures of *Capacity* commonly used in *England* are of two sorts, either *wet* or *dry*, for measuring of *Liquids*, as Beer, Syder, Wine, &c. (we take it for granted that) there are *two* distinct Gallons and a *third* for *dry* Commodities, as Corn &c. the Gallon for Beer and Ale, contains 282, for Wine 231, and for Corn 272 $\frac{1}{4}$ Solid Inches.

See these more fully explain'd in the following Tables, each of which doth shew how many Solid Inches are contained in any of the other Measures expressed therein.

A Table of Beer Measure.

Inches.						
35$\frac{1}{4}$	*Pints.*					
70$\frac{1}{2}$	2	*Quarts.*				
282	8	4	*Gallons.*			
2538	72	36	9	*Firkings.*		
5076	144	72	18	2	*Kilderkins.*	
10152	288	144	36	4	2	*Barrel.*

Inches.

A Table of Ale Measure.

Inches.						
$35\frac{1}{4}$	*Pints.*					
$70\frac{1}{2}$	2	*Quarts.*				
282	8	4	*Gallons.*			
2256	64	32	8	*Firkings.*		
4512	128	64	16	2	*Kilderkins.*	
9024	156	128	32	4	2	*Barrel.*

A Table of Wine Measure.

Inches.							
$28\frac{7}{8}$	*Pint.*						
$57\frac{3}{4}$	2	*Quart.*					
231	8	4	*Gallon.*				
4158	144	72	18	*Rundlet.*			
14553	504	252	63	$3\frac{1}{2}$	*Hogshead.*		
19404	672	336	84	$4\frac{2}{3}$	$1\frac{1}{3}$	*Tercion.*	
29106	1008	504	126	7	2	$1\frac{1}{2}$	*Pipe.*
58212	2016	1008	252	14	4	3	2 *Tun.*

This Table shews that in 1 *Tun* there is 2 *Pipes*, 3 *Tercions*, 4 *Hogsheads*, 14 *Rundlets*, 252 *Gallons*, 1008 *Quarts*, 2016 *Pints*, and 58212 Solid *Inches*. The Tables for *Beer* and *Ale* are like this, and need no explanation.

Sect.

Sect. III.

The Uſe of the Rule in Meaſuring of Superficies, and firſt of a Circle.

THE Area or Superficial Content of a Circle is found by knowledge of the Diameter or Circumference, I ſhall therefore firſt ſhew how by either of theſe to find the other.

Problem I.

The Diameter or Circumference of a Circle either being given to find the other.

THE Circumference of that Circle whoſe Diameter is Unity (or 1) is * 3.1415926536; but for our purpoſe 3.141592, will ſuffice: Therefore, as 1 is to 3.141592, ſo is the Diameter of any Circle to the Circumferenee: By the Inſtrument thus,

Set 1 on the Line A, againſt 3.141592 on the Line B, this done againſt any Diameter on the Line A, you have the Circumference on the Line B, and the contrary thus: Againſt,

* Foſter's *Problemata Geomet. Varia. Propoſ.* 2.

Theſe

These Diameters.	20	you have these Circumferences.	62.831
	30		94.247
	40		125.663
	50		157.079

Or Contrariwise against,

These Circumferences.	20	you have these Diameters.	6.366
	30		9.549
	40		12,732
	50		15,915

And so of any other.

Problem II.

The Diameter of any Circle being given to find the Area (or any part thereof) in Inches, in Ale or Wine Gallons, and also in Ale or Beer Barrels.

1. *For the whole Area in Inches.*

THE Area of a Circle is equal to the Fact or Rect-Angle of half the Diameter into half the Circumference, that is to say, if half the Diameter be Multiplied by half the Circumference, the Product will be the Area.

Thus, when the Diameter is 1, the Circumference is 3.141592, the half of this is 1.570796, which Multiplied by half the Diameter, (*viz.* 5.) the Product will be the Area of that Circle whose Diameter is 1, (*viz.*) .785398.

The

The Areas of all Circles are in proportion one to another, as the Squares of their Diameters, (2.12. of *Euclid.*)

Therefore, as the Square of the Diameter of any Circle is to the Area of that Circle;

So is the Square of the Diameter of any other Circle to the Area thereof.

In the Circle above mentioned the Diameter is 1, and the Area .785398: Now the Square of 1 being but 1 it muſt hold: *As* 1 is to .785398, *So* is the Square of any Diameter to the Area: So .785398 is a fixed Multiplicator, and if an Unit with Cyphers be divided by .785398 the Quotient will be 1.27324 a fixed Diviſor, and by either of theſe fixed Numbers the Area of any Circle may be found, either by Multiplication or Diviſion, if the Diameter be firſt given. For if the Square of any Diameter be

Multiplied	by	.785398	Prod.	is
Divided	by	1.27324	Quot.	is

the Area in Square Inches, Feet or Yards, according as the Diameter was meaſured by Inches, *&c.*

But with more expedition by the Inſtrument, Thus,

Set 1 (a Diameter) upon the Line D, to .785398 (the Area thereof) upon C.

This done, the Lines are like a Table of Circles Areas to all Diameters, for againſt any Diameter upon the Line D, you have the Area thereof upon C.

Example: *Let the Diameter be* 20.

Set 1 upon D, to .785398 upon C, and then againſt 20 upon D, is 314.159 the Area required upon C, and as the Rule now ſtands I alſo find that when the

The Diameter is { 25, 30, 40 } the Area will be { 490.87, 706.85, 1256.63 } Inches.

Contrariwise when the

Area is { 300, 400, 500 } the Diam. will be { 19.54, 22.56, 25.23 }

2. For the Area in Gallons or Barrels.

The Area in Inches Divided by 282 or 231, Quote the Area in Ale or Wine Gallons respectively, and so for any other Measure expressed in the former Tables, but without knowing the Area in Inches, the Area in Gallons or Barrels may be found thus: Divide .785398

By { 282, 231, 9024, 10152 } the Quotient is { .0027851 A G, .0033999 W G, .00008703 A B, .00007736 B B }

The several Quotients are the Area of a Circle whose Diameter is 1, in Ale or Wine Gallons, Ale or Beer Barrels respectively, and are fixed Multiplicators for finding the Area of all Circles in any of the Measures above named, for if the Square of the Diameter of any Circle be Multiplied by any of these Numbers, the Product is the Area in Ale or Wine Gallons, *&c.* respectively.

If you would effect this by Division, the several Divisors are thus found, Multiply the Divisor for finding the Area in Inches, *viz.* 1.27324

By

By	282	the Product is	359.05	A G
	231		294.11	W G
	9024		11489.71	A B
	10152		12925.93	B B

These several Products are the Divisors sought. And the Square of the Diameter of any Circle divided by one of these, quotes the respective Area.

Thus by the Pen.

But the Area of any Circle may be more readily found by the help of certain fixt Numbers called Gage-Points, and these fixt Numbers are the Diameters of those Circles whose Content at one Inch deep is equal to the respective Gallons or Barrels to which they belong, thus the Gage-Point for the Ale-Gallon is 18.95 which is the Diameter of that Circle, whose Content is 282 the Square Inches in the Ale Gallon.

The several Gage-Points are the Square-Roots of the Divisors last mentioned, and by the Rule are all found at once by the Lines C and D, for (setting these Lines even at the end,)

Against	359.05	are these Gage-Points upon D.	18.95	
these	294.11		17.15	
Divisors	11489.71		107.19	
upon C.	12925.93		113.69	

To which are set the Letters *a b. b b. w g.* and *a g.*

Now by these Gage-Points the Area of any Circle may be thus found.

For

For Ale Gallons.

Set 18.95 (the Gage-Point for Ale Gallon) upon D, to 1 upon C, then againſt any Diameter upon D you have the Area upon C.

So if the Diameter were 40 Inches, the Area will be 4.45 Ale Gallons, and the Rule being thus ſet, the Lines are like a Table of Circles Areas, for I likewiſe find that if the Diameter

Be	45 50 52.5 58.6	the Area will be	5.64 6.96 7.67 9 56	A G

What hath been ſaid of finding the Area of a Circle in Ale Gallons, may be underſtood of Wine Gallons, and alſo of Ale or Beer Barrels, obſerving to uſe the proper Gage-Points.

Note, When the Area of any Circle is ſought in Ale or Wine Gallons, if the Diameter be more than 18.95 yet leſs than 100, ſet the Gage-Point upon D, to 1 at the beginning of C, then againſt any Diameter from the Gage Point to 100 upon D, you have the Area upon C. Thus the Rule being ſet for Ale, you will find that when the

Diamet. is	30 40 60 80 100	the Area will be	2.507 4.456 10.026 17.825 27.851	Gallons.

When the Diameter is leſs than the Gage-Point or more than 100, then ſet the Gage-Point

Point to 1 in the middle upon C, this done againſt theſe

Diameters upon D	15	you have theſe Area's upon C	.627	AG
	13		.471	
	10		.278	

And without moving the Rule, if 10 at the beginning of D be 100, the Area againſt it will be 27.851: So againſt this Diameter, *viz.* 200, the Area is 111.4 Gallons, againſt 300, it is 250.7, and ſo on to 600 Inches Diameter, againſt which you have 1002.9, *&c.* by this you may further obſerve, that if the Diameter be increaſed by Tens, the Area will increaſe by Hundreds: thus,

the Diameter	10	gives	.27851
	100		27.851
	60		10.026
	600		1002.6

Gallons.

The like for Wine Gallons.

Laſtly.

To find any part of the Area of a Circle in Ale or Wine Gallons, &c.

Set the Gage-Point to $\frac{1}{3}$, $\frac{1}{2}$ or any other part of 1; then againſt the Diameter you have the like part of the Area.

Example. *Let it be required to find the third part of the Area of a Circle in Ale Gallons.*

Set the Gage-Point to $\frac{1}{3}$ of 1, *viz.* .333, then a-

gainſt any Diameter you have $\frac{1}{3}$ of the Area, thus againſt 100 is 9.283, which is $\frac{1}{3}$ of 27.85, the whole Area.

Underſtand the like for Wine Gallons, Ale or Beer Barrels.

Problem III. Figure 2.

The two Diameters c d *and* e f *of an Ellipſis, being given to find the Area or Content in Ale Gallons.*

AS the Square of the Diameter of a Circle, is to the Area of that Circle; So is the Rect-Angle or Product of greater and leſſer Diameters of an Ellipſis to the Area thereof. Therefore Multiply the greater Diameter by the leſſer Diameter, then that Product Multiplied or Divided by the fixed Multiplicators or Diviſors given (in pages 46 and 47,) gives the Area in Inches, Gallons or Barrels, according to the Number made uſe of.

Or thus, (by Problem VIII. Sect. I.) find a Geometrical mean Proportion between the greater and leſſer Diameters, for this mean is the Diameter of a Circle whoſe Area is equal to the Area of the Ellipſis.

Example. Fig. 2.

Let the greater Diameter *c d* be 72 Inches, and the leſſer *e f* 50, by the Rule above cited, the Geometrical mean between them will be found to be 60, the Diameter of a Circle equal to the Ellipſis, and the Area of a Circle, whoſe Diameter is 60, will be found to be 10.02 Ale Gallons. But the Area of an Ellipſis may be more eaſily found by the Rule, thus;

Set

Set 359,05 upon B to one of the Diameters (suppose 50) upon A; then against the other Diameter (suppose 72) upon B, you have the Area upon A, which in this Example will be 10.02, Ale Gallons, the Content of this Ellipsis at one Inch deep: the like may be done for Wine Gallons, Ale or Beer Barrels by taking the respective Numbers laid down in page 47.

Problem IV.

To find the Area or Content of a Triangular Superficies.

IN all Right-lined Triangles, Multiply half the longest side (or Base) by the Perpendicular (which is always the nearest distance from the Base to the opposite Angle;) or Multiply half the Perpendicular by the whole Base; this done, the Product is the Area required.

Example.

Let Fig. 3. represent a Triangular Back or Cooler, whose longest side *a b*, is 260 Inches, and the Perpendicular Line *c o* 110 Inches, the half of *a b* is 130, this Multiplied by 110 gives 14300 for the Area in Square Inches, and this divided by 282 quotes 50.7 the Area in Ale Gallons, or by the Rule at one operation thus:

Set 282 upon B to 130 upon A, then against 110 upon B, is 50.7 upon A, the Content as before.

To meaſure Quadrangular Figures.

Problem V.

In any Right-angled Parallelogram be it Square or Oblong: The Product of any two ſides including one and the ſame Angle, is equal to the Area or Content.

1. Example.

IN Fig. 5. the ſides *h i* and *i k* are equal, ſuppoſe each be 138 Inches; this Multiplied by it ſelf is 19044 the Area in Square Inches: For the Area in Gallons by the Rule:

Set 282 upon B, to 138 upon A, then againſt 138 upon B, is 67.53 the Area in Ale Gallons at one Inch deep.

2. Example. *Of an Oblong* (*Fig.* 4.)

Suppoſe *n s* 130 Inches, and *s u* 180, the Product of theſe (being Multiplied) is 23400 the Area in Inches: but for Ale Gallons, ſet 282 upon B, to 180 upon A, then againſt 130 upon B, is 82.9 Ale Gallons the Area required.

Problem

Problem VI.

In any Oblique Angled Parallelogram (be it a Rhombus or a Rhomboides) Multiply the ſhorteſt diſtance between the two longeſt ſides by one of the ſaid ſides, the Product is the Area.

1. Example. *Of a Rhombus (Fig. 7.)*

Suppoſe the ſide *p t* were 130 Inches, and the diſtance *q y* 108, theſe Multiplied the Product will be 14040 the Area in Inches: But by the Rule thus,

As 282 is to 108, *So* is 130 to 49.78, the Area in Ale Gallons at one Inch deep.

2. Example. *Of a Rhomboides (Fig. 6.)*

Let the ſide *x z* be 260, and the diſtance *t w* 108, the Product of theſe is 28080 the Area in Inches. By the Rule:

Set 282 upon A, to 260 upon B, then againſt 108 upon A, is 99 57 the Area in Ale Gallons.

Problem VII. Figure 8.

To find the Area or Content of a Trapezium.

A Right Line drawn from one of the Acute Angles to the Angle oppoſite, (as the Line *a b*) will divide the Trapezium into two Triangles, the

Area's of which are equal to the whole Trapezium, and may be found by the 4. Problem of this Section: Or thus, Multiply the sum of the two Perpendiculars (*fe* and *cr*) by the half of *ab*, the Product is the Area of the Trapezium, this is so plain that it needs no Example.

All other irregular Right-lined Figures, consisting of more than four sides, must be divided into Triangles, (which will ever be less by two than the Number of sides) and then the Area's of all those Triangles are equal to the Area of the whole Figure.

Problem VIII.

The sides of any Regular Polygon being given to find the Area.

IN any Regular Polygon, Multiply half the Sum of the sides by the Perpendicular (or neareſt diſtance from the Center to one of the sides) this done the Product is the Area.

Example, in a Pentagon (or Figure of five equal sides;) as Fig. 9.

Suppose each side be one Inch, to find the Perpendicular (C A) we have given A B = .5, and the Angle at C = 36 *d.*

Therefore,

	Log.
As the *fine* C = 36 *d.*	*Co. Ar.* 0.230781
Is to the side A B = .5	0.698970
So is the *fine* B = 54. *d.*	9.907957
To the side C A = 68819	.837708

which

which is the Perpendicular sought, and this .68819 Multiplied by half the sum of the sides, *viz.* 2.5, the Product is 1.7207, the Area of a Pentagon whose side is 1 Inch; and by this method I find the Area of all the other Polygons which are expreſt in the following Table. In the first Column of which you have the names of the Polygons, in the second their Area's in Inches, each of which being divided by 282 quotes the Area in Ale Gallons, which are the Numbers in the third Column.

Names of Polygons.	*Areas in Inches.*	*Areas in Ale Gall.*
Pentagon	1.72047	.006101
Hexagon	2.59809	.009212
Heptagon	3.63440	.012838
Octagon	4.82840	.017120
Nonogon	6.18210	.021920
Decagon	7.69400	.027280
Undecagon	9.36760	.033210
Dodecagon	11.1960	.039700

Now having the Area (both in Inches and Ale Gallons) of these Polygons when the side is Unity, the Area of each may be readily found when the side is any known length: For,

As the Square of the side of any Polygon is to the Area of that Polygon;

So is the Square of the side of any other like Polygon to the Area thereof:

Therefore,

If the Area of any Polygon (expreſſed in the Table) be desired.

Multiply the Square of the ſide by a Number in the ſecond Column againſt the name of the Polygon given, the Product will be the Area in Square Inches,

And if the Square of the ſide taken in Inches be Multiplied by a Number in the third Column, the Product is the Area in Ale Gallons.

Example.

Suppoſe the ſide of a Pentagon be 50 Inches, this Squared is 2500, which Multiplied by the firſt Number in the ſecond Column, (*viz.*) 1.72047, the Product will be 4301.175, the Area in Square Inches.

And if the Square of 50 (*viz.*) 2500 be Multiplied by the firſt Number in the third Column, *viz.* .006101, the Product will be 15.252 the Area in Ale Gallons, which agrees exactly with the former, for if you divide the Area in Inches, *viz.* 4301.175 by 282, the Quotient will be 15.252.

This may be eaſily performed by the Inſtrument, for there you have the Square of any Number by Inſpection, and the Multiplication is quickly wrought by the Lines A and B.

But the Lines C and D, will effect the whole with much more expedition, for at once ſetting of the Rule you have the Area (in Inches or Ale Gallons) to any given ſide, or the ſide to any Area given.

Example.

For the Area in Ale Gallons, Set 1 upon D to (the Area in Gallons when the ſide is 1, *viz.*) .006101 upon C, this done, againſt any ſide upon D you have the Area upon C, and the contrary.

Thus

Thus against 50 upon D, is 15.252, the Area in Ale Gallons: Also against 22 Gallons an Area upon C, you have 60 the side upon D, *&c.*

It is needless to give Examples of the other Polygons, for what hath been said of the Pentagon may be understood of the rest, remembring to use the proper Numbers as aforesaid.

SECT. IV.

The Use of the Rule in the Mensuration of Solids, and first of a Prism.

A *Prism* is a Solid contained by several Planes, two of which being opposite are called the Bases, and these are equal, Parallel, alike and alike situate, but the other Planes are Parallelograms, in which a Right Line may be every where applied from Base to Base: Under this name *Prism* is comprehended that Solid of two Circular Bases, usually called a Cylinder.

Problem I.

To find the Solid Content of a Prism.

MUltiply the Area of the Base by the Perpendicular height, Product is the Content sought.

Example.

Let (Fig. 11.) represent a Brewers Tun in the form of a Square Prism, whose Base *g, h, k, i,* is equal to Fig. 5. each side being 138 Inches, and the Perpendicular (*s i*) 30 Inches, how many Ale Gallons or Beer Barrels will this Tun contain?

Set 282 upon B to 138 upon A, then against 138 upon B is 67.534 upon A, the Content in Ale Gallons at one Inch deep, this Multiplied by 30, the whole depth gives 2026.02 the whole content in Ale Gallons, this divided by 36 quotes 56.283 the Content in Beer Barrels.

The Fraction .283 (being reduced) is 10 Gallons and .188 parts. For

As 1.000 : 36 :: .283 : 10.188.

2. Example. *Of a Round Prism or Cylinder.*

Let Fig. 12. Rrepresent a round Tun, whose Diameter (*n o*) at top is equal to (*s y*) the Diameter at bottom, each being 120 Inches, and the Altitude *e x* 36 Inches, How many Gallons or Beer Barrels may this Tun contain? First for Gallons.

Set the Gage-Point (*s g*) to 36 the Tuns depth upon C, then against 120 the Diameter upon D, is 1443.6 the Content in Ale Gallons.

2. For the Content in Beer Barrels: 1443.6. Divided by 36, quotes 40.1 the Content in Barrels.

Or rather thus,

Set the Gage-Point for a Beer Barrel, to 36 the depth upon C, then against 120 upon D, is 40.1. the Content as before, and so for any other depth, as,

Suppose

Suppose I come to this Tun and find the Liquor 9 Inches deep, How many Beer Barrels are then contained in the Tun?

Set the Gage-Point to 9, then against 120 is 16.02, that is, 10 Barrels and 1 Gallon *fere*, the quantity of Liquor sought.

Problem II.

The Diameter and depth in Inches, and the Content in Beer Barrels of any Cylindrical Tun, any two being given the third may be found.

IN this Problem are Three Questions, but all resolved at once setting the Rule.

1. By the depth and Content to find the Diameter.

Example.

Suppose the depth 40 *Inches, and the Content* 50 *Beer Barrels, What is the Diameter?*

Set 40 the depth upon C, to the Gage-Point upon D: Then against 50 the Content upon C is 127.1 Inches, the Diameter sought.

2. By the Diameter and Content to find the depth, without moving the Rule: Say,

As 127.1 the Diameter is to 50 the Content.
So is the Gage-Point to 40 the depth } upon { D C D C

3. By

3. By the Depth and Diameter to find the Content, the Rule ſtanding as before: Say,

As the Gage-Point is to 40 the depth.
So is 127.1 the Diameter to 50 the Content.
} upon { D / C / D / C }

Note, As a Circle is the Baſe of a Cylinder or round Priſm, ſo a Triangle, Quadrangle, or any other plain Superficies may repreſent the Baſe of a Solid, for if there be Planes erected perpendicularly upon the Line or Lines, which incompaſs any ſuch Superficies, they will generate a Solid which may be called a Priſm, and the Content of any ſuch Solid is gotten by Multiplying the Area of the Baſe by the Altitude or diſtance from one Baſe to another.

SECT.

Sect. V.

The Use of the Rule in the Menſuration of a Sphere, and its Fruſtums.

Problem I.

The Diameter of a Sphere or Globe being given in Inches, to find the Content thereof in Inches or Gallons both Ale and Wine.

A Sphere is two-third parts of a Cylinder, whoſe Diameter and Altitude are equal to the Diameter of the Sphere;

But if the Diameter and Altitude of a Cylinder be 1, the Area of the Baſe (which is alſo the Solid Content (for 1 doth not Multiply) is .785398: Therefore $\frac{2}{3}$ of .785398, *viz.* .523598 is the Content of a Sphere whoſe Diameter is Unity or 1.

Moreover, all Bodies are in proportion one to another as the Cube of their like ſides:

Therefore,

As the Cube of the Diameter of any Sphere is to the Content of that Sphere;

So is the Cube of the Diameter of any other Sphere to the Content thereof.

Now if the Diameter be 1, the Cube thereof is but 1.

Therefore,

Therefore,

As 1 is to .523598:
So the Cube of the Diameter of any Sphere to the Content.

Example. *Let the Diameter of a Sphere be* 20 *Inches, and the Content required in Solid Inches.*

By the Lines D and C on the Rule, the Cube of 20 will appear to be 8000, this Multiplied by .523598, is 4188.78 the Content sought.

For the Content in Gallons.

If .523598 be divided
By { 282 / 231 } quotes { .001856 / .002266 } the content in { A. G. / W. G. }

Now if the Cube of the Diameter of any Sphere be Multiplied by either of these Numbers, the Product will be the Content in Ale or Wine Gallons respectively: Thus, The Cube of 20 the Diameter (*viz.*) 8000 Multiplied by .001856 is 14.848 the Content in Ale Gallons, and the like may be done for Wine.

But the Content of any Sphere may be more readily found by the Instrument.

Thus,

Set 1 upon D, to .523598 upon E, then against any Diameter upon D, is the Content upon E: So against 20 (the Diameter above-mentioned) upon D, is 4188.78 the Content in Inches as before: In like manner if the Diameter were 30, the Content will be 14137.14, *&c.*

For

For Ale Gallons thus,

Set 1 upon D, to .001856 upon E, then againſt any Diameter upon D, is the Content upon E, as againſt 20 upon D is 14.84, the Content as before, and without moving the Rule I find that if the

Diameter be { 30 / 40 } the content is { 50.13 / 118.78 } A. G.

Problem II.

Having the Altitude of the Fruſtum of a Globe, together with the Diameter of its Baſe, to find the Altitude of the other Fruſtum or (which is all one) the remainder of the Globes Axis.

The Rule is,

DIvide the Square of the Semidiameter of the Fruſtums Baſe, by the Altitude of either of the Fruſtums, the Quotient will be the Altitude of the other Fruſtum.

Example.

In Fig. 10. Let *c b* the Altitude of the Fruſtum be 6 Inches, and *d c e* the Diameter at the Baſe 24, the half of this (*viz. d c*) is 12, which Squared is 144, this Divided by the Altitude *b c*, *viz.* 6, the Quotient is 24 equal to *c a*, the Altitude of the other Fruſtum. Note alſo, that a Geometrical mean proportion between *a c* and *c b*, is equal to *c d*, which doubled is *d e*, the Diameter of the Fruſtums Baſe.

Problem

Problem III.

Having the Altitude of the greater and leſſer Fruſtums of a Globe, and the Diameter of the Baſe, to find the Content of the leſſer Fruſtum.

The Rule.

1. BY the Diameter at the Fruſtums Baſe find the Area of a Circle equal thereto, which Multiply by the leſſer Fruſtums Altitude reſerving the Product.

2. To $\frac{1}{2}$ the Altitude of the greater Fruſtum, add $\frac{1}{6}$ of the Altitude of the leſſer; this done, Multiply the Sum by the former Product, and divide this laſt Product by the Altitude of the greater Fruſtum, the Quotient will be the Content of the leſſer Fruſtum ſought.

Example. In Fig. 10. Let *c a*, the Altitude of the greater Fruſtum, and *d c e* the Diameter of the Fruſtums Baſe be each 24 Inches, *c b* the Altitude of the leſſer Fruſtum 6 Inches, What is the Content of the leſſer Fruſtum (*d c e b*) in Ale Gallons?

The Area of a Circle whoſe Diameter is equal *d e* (*viz.* 24 Inches) is 1.6042, this Multiplied by *c b*, 6, is 9.6252 which keep: Again, half *a c* is 12, to which add $\frac{1}{6}$ of *c b*, *viz.* 1, the Sum is 13, this Multiplied by 9.6252 is 125.127, and this divided by *a c*, *viz.* 24, the Quotient will be 5.213. the Content of the Fruſtum in Ale Gallons. But this may be perform'd with greater expedition by the Inſtrument.

Thus,

Thus,

1. Set 1 upon D to the Fruſtums Altitude, *viz.* 6, upon C, then againſt the Diameter of the Baſe, *viz.* 24, is 9.625, which keep.
2. Set the Altitude of the greater Fruſtum, *viz.* 24, upon A, to 13 (that is $\frac{1}{2}$ *a c*, more $\frac{1}{12}$ of *c b*) upon B, then againſt 9.6252 upon A is 5.2134, the Content as before.

SECT. VI.

The Uſe of the Inſtrument in the Menſuration of Pyramids and their Fruſtums.

A *Pyramid* is a Body contained under ſeveral Planes, ſet upon one Right-Lined Baſe, and meeting in a point at the top which is called the Vertex, alſo in each of theſe Planes a Right-Line may be every where applied from the Baſe to the Vertex.

And under this name *Pyramid* is Comprehended that Pyramidical Body (whoſe Baſe is a Circle) commonly called a Cone.

Problem

Problem I.

To find the Solid Content of a Pyramid.

A *Pyramid* is one third part of a *Prism*, which hath the same Base as the Pyramid, and the Altitude equal thereto:

Therefore,

Multiply the Area of the Base by $\frac{1}{3}$ of the Altitude, or $\frac{1}{3}$ of the Area by the whole Altitude, in either cases the Product will be the Content of the Pyramid.

This is universal for all Pyramids in what ever form their Bases be, and the Content thus found will be Inches, Gallons, Barrels, *&c.* according as the Area of the Base was given in any of these Measures.

1. Example.

In a Square Pyramid (as Fig. 17.) suppose the side of the Base *x p* be 112 Inches, and the Altitude *x r* 90 Inches, what is the Content in Ale Gallons, or Beer Barrels?

Set 282	} upon }	to 112	} upon
then against 112	} B {	is 44.48	} A,

the Area of the Base in Ale Gallons. Then, Set 1 upon A, to 30 upon B, and against 44.48 upon A is 1334.46 the Content in Ale Gallons.

For

For the Content in Beer Barrels.

Set 10152 (the Inches in a Beer Barrel) upon B to 112 upon A; then againſt 112 upon B is 1.2356, the Area of the Baſe in Beer Barrels.

Again,

Set 1 upon B, to 30 is 37.06 upon A.
then againſt 1.235

The Content is 37 Barrels and 2 ½ Gall. ferè.

2. Example.

There is a round Pyramid or Cone (as Fig. 18.) ſuppoſe the Diameter at the Baſe *a n o* be 100 Inches, and the Altitude *n x* 180; How many Ale Gallons, or Beer Barrels may this Cone contain.

For Ale Gallons.

Set the Gage-Point for an Ale Gallon to ⅓ of the Altitude *(viz.* 60) upon C: then againſt 100 upon D is 1671 the Content required.

For the Content in Barrels.

Set the Gage-Point for a Beer Barrel *(viz.*113.69) to 60 upon C: then againſt 100 upon D is 46.417 the Content ſought; the Fraction .417 is equal to 15 Gallons: For,

As 1 to 36:
So is .417 to 15;
So the whole Content is 46 Barrels and 15 Gallons.

Problem

Problem II.

To find the Solid Content of the Fruſtum of a Pyramid.

1. BY the Rules aforegoing, find the Area of the greater, and alſo of the leſſer Baſe.

2. Find a Geometrical mean Proportion betwixt theſe two Areas.

3. The ſum of theſe three Numbers being multiplied by one third part of the Altitude, the Product is the Content of the Fruſtum ſought.

Example. Let *a c e o* (in Fig. 18.) repreſent the Fruſtum of a Round Pyramid or Cone, *a o* the Diameter of the greater Baſe is 108 Inches, *c e* the Diameter of the leſſer Baſe 81, and *n v* the Altitude 36 Inches, to find the Content in Ale Gallons.

1. Set the Gage-Point to 1 in the middle of the Line C; then againſt 108 upon D is 32.48, the Area of the greater Baſe in Ale Gallons.

2. Set the Gage-Point to 1 at the beginning of the Line C; then againſt 81 upon D, is 18.27, the Area of the leſſer Baſe.

3. Set 32.48 upon C, to 32.48 upon D; then againſt 18.27 upon C, is 24.36 the Geometrical mean between the two Area's.

So the Area of the greater Baſe is	32.48
The Area of the leſſer is	18.27
The Geometrical mean betwixt the two Area's is	24.36
The ſum is	75.11

This multiplied by $\frac{1}{3}$ of the Altitude (*viz.* 12) gives 901.32 the Content of the Fruſtum ſought.

SECT.

SECT. VII.

The Use of the Rule in Gaging of Brewers Tuns.

THese Vessels are in several Forms: the most usual may be considered under some of these varieties,

I. *Such whose Bases are equal.*

And if the equal Bases be alike (that is both Round, both Square, &c.) and Parallel, and the sides of the Tun straight from one Base to the other, these Tuns are called Prisms. See Problem I. of this Sect.

II. *Such whose Bases are unequal.*

1. If the unequal Bases be alike (suppose both Rectangular Parallelograms, both Elliptical, &c.) Parallel, Proportional, and alike situate, and the sides of the Tun straight, such Tuns are the Frustums of Pyramids. See Problem II. of this Sect.

2. If the unequal Bases be unlike, suppose one Round, and the other Elliptical, or if they be alike but not Proportional, or if they be both alike and Proportional, yet if they be not alike situate, the Tun is called a Prismoid, in which 'tis still supposed the Bases are Parallel, and sides of the Tun straight. See Problem VII.

Note.

Note, When the Baſes are Proportional it will be,

As the length of the greater Baſe is to the length of the leſſer;

So is the breadth of the greater Baſe to the breadth of the leſſer.

And the Baſes are ſaid to be alike ſituate, when the length of the greater and leſſer Baſes are both in the ſame Plain.

Problem I. Figure 11.

There is a Tun whoſe Baſes are both equal and Square, each ſide of either Baſe being 138 *Inches, and the depth* 33 *Inches, what is the Content of this Tun in Ale Gallons?* Anſwer 2228.62.

For,

SET 282 upon B to 138 upon A; then againſt 138 upon B, you have 67.534 upon A, the Content at one Inch deep; this multiplied by the whole depth, *viz.* 33 gives 2228.62 the Content required.

Or thus;

Set 16.79 upon D to 33 (the Tuns depth) upon C, then againſt 138 upon D, is 2228.62 upon C, the Content as before.

Note, 16.79 is the Gage-Point for Square Veſſels, it being the Square-Root of 282 the Inches in an Ale Gallon.

It

It is needless to give more Examples of this sort of Tuns, for let the Bases be in any form whatsoever, if the Tun have the qualification above mentioned, the Content is found by Multiplying the Area of the Base by the depth; and if the Bases are Round, as in most Tuns they are, the Content may be found at one Operation; as hath been shewn in the I. and II. Problem of Sect. IV.

Of Tuns whose Bases are unequal.

Problem II. Figure 16.

There is a Tun, as a, b, c, d, e, f, g, h, *whose Bases are Rectangular Parallelograms, unequal but Parallel, and alike situate, and the sides Proportional,* (*therefore called the Frustum of a Pyramid*) d a *the length of the greater Base is* 100 *Inches, and* d c *the breadth* 80; h e *the length of the lesser Base* 85, *and* h g *the breadth* 68, *and the depth of the Tun* 30 *Inches: how may Ale Gallons may this Tun contain?*

FOR all Tuns of this sort, this is a general Rule.

Take the Area of the greater Base, the Area of the lesser Base, and a Geometrical mean Proportional betwixt the two Area's; the sum of these three multiplied by one third part of the Tuns depth, gives the Content.

Example.

Example either by the Pen, or the Lines A and B upon the Rule.

As 282 is to *d a* 100. *So* is *d e* 80 to the Area of the greater Base, *viz.*	28.368
As 282 is to *h e* 85. *So* is *b g* 68 to the Area of the lesser Base, *viz.*	20.496
A Geometrical mean between these two Area's will be found (by Prob VIII. Sect. 1.) to be	24.112
The Sum is	72.976

This multiplied by one third part of the Tuns depth, *viz.* 10, gives 729.76 the Content in Ale Gallons.

Problem III. Figure 16.

There is a Tun whose Bases (i l k m *the greater, and* n q o p *the lesser*) *are both Elliptical, Parallel, Proportional and alike situate; the greater and lesser Diameters of each Base being equal to the lengths and breadths of the Bases of the former Tun, and the depth the same; how many Ale Gallons will this Tun contain?*

THis Elliptical Tun may be conceived to be inscribed in the Tun, whose Content was found in the last Problem; all the given dimensions are the same in both, and the Operation will be the same too,

too, if for the first term in the Proportion we take 359.05 instead of 282: See the Work.

As 359.05 is to *i k* 100. *So* is *l m* 80 to the Area of the greater Base. —— } 22.281

As 359.05 is to *n o* 85. *So* is *p q* 68 to the Area of the lesser Base. —— } 16.098

The Geometrical mean between these two Area's will be found (by Problem VIII. Sect. I.) to be. —— } 18.938

The Sum is —— 57.317

This Multiplied by one third part of the depth, *viz.* 10 gives 573.17 the Content of this Elliptical Tun, in Ale Gallons.

Problem IV.

There is a Tun (as Fig. 17.) *whose Bases are both Square but unequal,* x p *the side of the greater Base is* 108 *Inches,* r o *the side of the lesser Base* 93, *and the depth* s a 30 *Inches, what is the Content in Ale Gallons?*

THis Tun being the Frustum of a Pyramid may be Gaged by the General Rule in Problem II. of this Section.

Or thus,

1. Square the side of the greater Base, and also the side of the lesser Base.

2. Multiply the greater side by the lesser, and to the Product add the two former Squares.

3. The Sum of these three being Multiplied by $\frac{1}{3}$ of the depth, gives the Content in Square Inches, which divided by 282 the Quotient is the Content in Ale Gallons.

Example.

By the Instrument the Square of *x p* 108 is	11664
The Square of *o r* 93, is	8649
108 Multiplied by 93, is	10044
Sum is	30357
Multiplied by $\frac{1}{3}$ Altitude	10
The Product, *viz.*	303570

This Divided by 282 quotes 1076.48 the Content sought:

Problem

Problem V. Figure 18.

There is a Conical Tun whose Bases are both Circular, a o *the Diameter of the greater Base is* 108 *Inches,* c e *the Diameter of the lesser Base* 93. *and* u n *the depth* 30 *Inches: what is the Content of this Tun in Ale Gallons?*

THE Dimensions being the same as in Problem IV. of this Section, proceed as is there directed, the last Product will be 303570, this divided by 359.05 quotes 845.48 the Content sought.

Or thus by the Rule.

Subtract the Diameter of the lesser Base, from the Diameter of the greater, and add half the difference to the lesser, the Sum is the Diameter in the middle of the Tuns depth; which found,

2. Set the Gage-Point (*a g*) to the Tuns depth upon C.

Then against the Diameter upon D, is a fourth number upon C, which keep.

2. Set the Gage-Point to $\frac{1}{3}$ of the Tuns depth upon C.

Then against $\frac{1}{2}$ the difference of the Diameters upon D, is a fourth Number, which added to the fourth Number first found, is the Content of the Tun sought.

Example. Figure 18.

a o the Diameter of the greater Base is —— 108
c e the Diameter of the lesser Base is —— 93

The difference is —— —— —— —— 15

The half difference is 7.5 this added to 93 makes 100.5 for the Diameter in the middle of the Tuns depth:

Therefore,

1. Set the Gage-Point to 30 the Tuns depth, then against 100.5 the Diameter is 843.91 which keep.

2. Set the Gage-Point to $\frac{1}{3}$ of the Tuns depth, that is 10; then against 7.5 the semidifference of the Diameters is 1.56 this added to 843.91 (the Number first found) the sum is 845.47 the Content sought, which agrees very well with the former Rule.

Problem VI.

To find the Content of a Pyramidal Tun whose Bases are in the form of any of the first 12 *Regular Polygons, one side of each Base, and the Tuns depth being given.*

THE Area of each Base of any such Tun may be found by Problem VIII. Section III.

A Geometrical mean betwixt these Area's may be found by Problem VIII. Section I.

The same of the two Area's and the Geometrical mean, being Multiplied by $\frac{1}{3}$ of the Tuns depth gives the Content.

Or

Or thus,

By the Rule in the laſt Problem, Square the ſide of each Baſe, Multiply the greater ſide by the leſſer; then Multiply the ſum of theſe three by $\frac{1}{3}$ of the Tuns depth, and Divide this laſt Product by a Number in the following Table, proper to the Polygon given, the Quotient will be the Content in Ale Gallons.

Example. Let there be a Pyramidal Tun, whoſe Baſes are both in the form of a Pentagon. (or figure of five equal ſides) and let the ſide of the greater Baſe be 108, the ſide of the leſſer 93, and the depth 30 Inches; all the ſame as in the laſt Problem: therefore according to the Rule and Example there laid down, the laſt Product will be 303570, this Divided by 163.91 (the proper Diviſor for a Pentagon) the Quotient will be 1852.05 the Content in Ale Gallons: and ſo for any other, taking the proper Diviſor out of this Table.

Names of Polygons.	*Diviſors for Ale Gallons.*	*Number of Sides.*
Trigon	651.25	III
Tetragon	282.00	IV
Pentagon	163.91	V
Hexagon	108.55	VI
Heptagon	77.59	VII
Octagon	58.41	VIII
Nonogon	45.62	IX
Decagon	36.65	X
Undecagon	30.11	XI
Dodecagon	25.18	XII
Cone	359.05	

To find the Content of any Tun whose Parallel Bases are Rectangular Parallelograms, or Ellipsis, the sides of the Tun being straight from the top to the bottom, whether the Bases be alike or unlike, Proportional or disproportional, alike situate or Inverted, by one General Rule.

Problem VII.

Let Fig. 16. *represent a Tun called a Prismoid, the Bases are Rectangular Parallelograms unequal and disproportional, but parallel and alike situate*; a d *the length below* = 150 *Inches, and* a b *the breadth* 100, e h *the length above* 90, *and* e f *the breadth* 70, *and the depth* 30: *What is the Content in Ale Gallons?*

The Rule.

1. TO *the greater length* a d *add* $\frac{1}{2}$ *the lesser length* e h, *and Multiply the sum by the greater breadth* a b, *reserving that Product.*

2. *To the lesser length* e h, *add* $\frac{1}{2}$ *the greater length* a d, *and Multiply the sum by the lesser breadth* e f; *add this Product to the former reserved Product.*

3. *Multiply the sum of these two Products by* $\frac{1}{3}$ *of the Tuns depth, and divide the last Product by* 282, *the Quotient will be the Content of the Tun in Ale Gallons.*

Example.

Example.

1. *a d* 150 added to ½ *e h* 45, the ſum is 195, this Multiplied by *a b* 100, the Product is 19500 which keep.

2. *e h* 90 added to ½ *a d* 75 the ſum is 165, this Multiplied by *e f* 70, the Product is 11550, this added to the former Product, *viz.* 19500 the ſum is 31050, this Multiplied by ⅓ of the depth, *viz.* 10, the Product is 310500, this Divided by 282 gives 1101.06 the Content in Ale Gallons.

The truth of this Rule will plainly appear if we duly conſider Fig. 13. in which *a b c d* is equal to the greater Baſe, and *e f g h* equal to the leſſer Baſe of the Tun in the laſt Problem, (which was repreſented by Fig. 16.)

In this Fig. 13. *b i o v* is made equal to *f e h g*.

Alſo, *a b* 100 leſs by *e f* 70, is equal to *a i* 30.

And *a d* 150 leſs *e h* 90, is equal to *s d* 60, the Altitude is 30 as before.

The Lines being drawn, the whole Solid is Compoſed of theſe parts, *viz.* three Priſms and a Pyramid.

The { 1, 2, 3 } Priſm is { *b i o u f e h g*; *i a s o h e*; *o v c n h g* }

The Pyramid is *o s n d h.*

The Content of theſe four Solids being added together, gives the Content of the whole Solid *a b c d e f g h*; for the whole is equal to all its parts taken together. (Axiom 19. 1. *Euclid.*)

And the Content of theſe may be found thus:

1. Multiply *f g* 90 by *f e* 70, the Product is 6300; this Multiplied by *b f* the Altitude, *viz.* 30, gives 189000 the Content of the Prism, *b i o v f e h g.*

2. Multiply *a i* 30 by *a s* 90, the Product is 2700, this Multiplied by $\frac{1}{2}$ the Altitude, *viz.* 15, is 40500 the Content of the Prism *i a s o h e.*

3. Multiply *e n* 70 by *n o* 60, the Product is 4200; this Multiplied by 15 is 63000, the Content of the Prism *o w c n h g.*

4. Multiply *s d* 60 by *s o* 30, the Product is 1800, this Multiplied by $\frac{1}{3}$ of the Altitude, *viz.* 10, gives 18000 the Content of the Pyramid *s o n d h.*

So the Content of the	1, 2, 3	Prism is 189000, 40500, 63000
Pyramid is		18000
The Sum		310500

Is the Content in Square Inches, and is exactly the same with the Content found by the Rule given in the last Problem.

The truth of this Rule being thus demonstrated, we shall shew its further usefulness in the following Problems.

Problem

Problem VIII. Figure 16.

There is a Tun (called a Cylindroid) whose Bases are both Elliptical, unequal, and disproportional (that is, there is not the same proportion betwixt the Axis and Diameter of the greater Base, as is betwixt the Axis and Diameter of the lesser) let the given Dimensions be the same as in the last Probl. viz.

$no = 90 : qp = 70$ } Rectangular } above.
$ik = 150 : lm = 100$ } Conjugates } below.

And the depth 30 *Inches: what is the Content of this Tun in Ale Gallons?*

For the Content of this Tun Work by the General Rule, and according to the Example given in the last Problem, only instead of 282 Divide the last Product by 359.05. So in this Example (the Numbers being the same as in the last) the last Product will be 310500 this Divide by 359.05 quotes 864.78 the Content in Ale Gallons.

Problem IX. Figure 15.

There is a Tun whose Bases are Rectangular Parallelograms and Parallel, but neither equal nor alike situate (for g h *the breadth above is posited directly opposite to* a d, *the length below) how many Ale Gallons will this Tun contain?*

The Dimensions are

h g Breadth = 68 : *h e* Length = 85 above
d a Length = 100 : *d c* Breadth = 80 below.
The Depth 30 *Inches.*

If the Bases of this Tun were alike situate (as is supposed in Problem II. of this Section) the Content would be 729.76 Ale Gallons as was there shewn; but the Bases being Inverted as abovesaid, the Content will be found to be 735.81 Ale Gallons, as appears by the following Work, which is performed by the General Rule laid down in Problem VII. of this Section.

Thus *h g* 68 added to $\frac{1}{2}$ *d a* 50 is 118, this Multiplied by *h e* 85 the Product is 10030.

Again *d a* 100 added to $\frac{1}{2}$ *h g* 34 is 134, this Multiplied by *d c* 80, the Product is 10720, the sum of these two Products is 20750, which Multiplied by $\frac{1}{3}$ of the depth, *viz.* 10, gives 207500 the Content in Square Inches, this Divided by 282, the Quotient is 735.81 the Content in Ale Gallons.

Or if the last Product be Divided by 359.05, the Quotient will be 577.91 the Content of an Elliptical Tun, whose Axis above is directly opposite to the

the ſhorter Diameter below; and ſuch an Elliptical Tun may be conceived to be inſcribed in the other, the reſpective Axi's and Diameters, Lengths and Breadths, and alſo the Depth being the ſame in both.

Problem X. Figure 14.

To find the Content of a Tun whoſe Baſes are unlike, ſuppoſe one Square, and the other Oblong; or one Round, and the other Elliptical, but Parallel one to the other, and the ſides of the Tun ſtraight.

The Dimenſions are,

Breadth.	Length.	
$ef = 74$:	$fg = 100$	} Rectangular } above
$dc = 100$:	$cb = 100$	} Conjugates } below.

Depth 30 *Inches.*

By the General Rule in Problem VIII. Work thus:

1. To ef 74 add $\frac{1}{2}$ dc 50 the ſum is 124, this Multiplied by fg 100 is 12400.

2. To dc 100 add $\frac{1}{2}$ ef 37 the ſum is 137, this Multiplied by cd 100 is 13700, this added to the former Product, *viz.* 12400, makes 26100, and this Multiplied by $\frac{1}{3}$ of the Depth, *viz.* 10, the Product is 261000, this Divided by 282 gives 929.53 the Content of this Tun in Ale Gallons, ſuppoſing the Baſes were one Square, and the other Oblong, as the Figure repreſents them:

But if the Baſes were one Round, and the other Elliptical, the laſt Product, *viz.* 261000 muſt be Divided

Divided by 359.05 and in this Case the Content will be 726.91.

By what hath been said I presume it will not be difficult to find the whole Content of any Tun, whose Bases are Parallel, and Sides straight from top to bottom.

Sect. VIII.

Shewing how to find the Content of any Tun from Inch to Inch, or what any Tun will contain upon every Inch of its depth, which is commonly called Inching of a Tun.

Problem I.

Suppose a Tun whose Bases are both Round, or both Square and Parallel one to another, and in the sides a straight Line may be every where applyed from Base to Base: let the Diameter or side of the greater Base be 187.2 *Inches, the Diameter or side of the lesser Base* 180 *and the depth* 12 *Inches, to Inch this Tun (be it Round or Square) from the lesser Base downward.*

Rule,

FRom the Side or Diameter of the greater Base Subtract the Side or Diameter of the lesser Base, Divide their difference by the Tuns depth, and call the Quotient *a*.

For the Side or Diameter of the lesser Base put *b*.

Mul-

Multiply the Number *b* by the Number *a*, and call the Product *c*. Then,

1. To the Square of *b* add the Number *c*, and one third part of the Square of *a*, the ſum of theſe three will ſhew the Content of the firſt Inch of the Tun.

2. Multiply one third part of the Square of *a* by 2, to this Product add the Number *c*, then Multiply this ſum by 2, and to this Product add the Square of *b*, Multiply this ſum by 2, then will this laſt Product ſhew the Content of the two firſt Inches of the Tun.

3. Multiply one third part of the Square of *a* by 3, then Add and Multiply as in the laſt, the third and laſt Product will ſhew the Content of the three firſt Inches of the Tun.

4. Having got theſe three Numbers, Subtract the firſt from the ſecond, and likewiſe the ſecond from the third, then the firſt Number, and each of theſe differences or remainders being ſeverally Divided by 282 for Square Tuns, or by 359.05 for Round Tuns, the reſpective Quotients will be the Content (in Ale Gallons) of the firſt, ſecond, and third Inches of the Tun, by which a Table may be made for the whole Tun as in the Example following.

Example of a Square Tun.

The ſide of the greater Baſe	187.2
The Side of the leſſer Baſe	180.0
The remainder is	7.2

This Divided by the Tuns depth, *viz.* 12 quotes .6 for the Number *a*,

The

The Number *b* (which is the Side of the leſſer Baſe) is 180, this Multiplied by the Number *a viz:* :6 gives 108 for the Number *c*.

1. The Square of D is	32400
Number *c* is	108.
$\frac{1}{3}$ Of the Square of *a* is	.12
The ſum is	32508.12

For the firſt Inch of the Tun.

2. $\frac{1}{3}$ Of the Square of *a* is	.12
Multiply by	2
	.24
Number *c* added	108.00
Sum is	108.24
Multiply by	2
	216.48
Square of Number *b* added	32400.00
Sum is	32616.48
Multiply by	2
Sum is	65232 96

For the two firſt Inches.

3. $\frac{1}{3}$ Of

3. $\frac{1}{3}$ Of the Square of *a*	.12
Multiply by	3
	.36
Number *c* add	108.00
Sum is	108.36
Multiply by	3
	325.08
Square of *b* added	32400.00
Sum is	32725.08
Multiply by	3
Sum is	98175.24

For the three first Inches.

Two first Inches	65232.96
First Inch	32508.12
Rest for second Inch	32724.84
Three first Inches	98175.24
Two first Inches	65232.96
Rest for third Inch	32942.28

Now

Now these three Numbers:

viz. { 32508.12 } Divided { 115.277
32724.84 } by 282 { 116.045
32942.28 } quotes { 116.816

For the Content (in Ale Gallons) upon the first, second, and third Inches of this Tun, supposing the Bases Square.

And if the said Numbers be Divided by 359.05 the respective Quotients will

Be { 90.539 } the Content of the first, second,
91.142 } and third Inches of the Tun, when
91.747 } its Bases are Round.

Now by these Numbers the following Table is Composed for the Round Tun. Thus,

Take the first Inch from the second, the remainder or difference is .603, which set down in the third Column, against the space betwixt the first and second Inches. In like manner take the second Inch from the third, the remainder is .605, which place in the third Column under the other, then take .603 from .605, there remains .002 for the second Difference, which is always the same as in the fourth Column, this second Difference added to the second Number in the third Column, *viz.* .605, makes .607, and this added to the Content of the third Inch, *viz.* 91.747 makes 92.354 the Content of the fourth Inch, and so by a continual addition of the second Difference to the first, and of their sum to the Inch last found, the Table is made.

Depth

Depth in Inches.	*Content of every Inch in Ale Gall.*	*First Differ.*	*Second Differ.*
1	90.539		
		.603	
2	91.142		.002
		.605	
3	91.747		.002
		.607	
4	92.354		.002
		.609	
5	92.963		.002
		.611	
6	93.574		.002
		.613	
7	94.187		.002
		.615	
8	94.802		.002
		.617	
9	95.419		.002
		.619	
10	96.038		.002
		.621	
11	96.659		.002
		.623	
12	97.282		
Sum--1126.706--Content of the Tun.			

In this Example we have made a Table for the Round Tun, and the Operation would have been the ſame for the Square Tun, had we taken the three firſt Inches of that in ſtead of theſe.

Note, When we Inch a Tun by this Rule, we ſuppoſe it to be Regular, having all the qualifications expreſt in this Problem. But for as much as moſt Tuns have ſome irregularity or unevenneſs in their ſides, I ſhall here lay down a more Practical Rule for Inching of Tuns.

Problem

Problem II.

There is a Tun whose Diameter at the bottom is 108 *Inches, the Diameter at the top* 93, *and the depth* 30, *how many Ale Gallons will this Tun contain upon every Inch of the depth?*

TO resolve this Question, there must first be known the whole Content of the Tun, and also the Area of the mean Diameter in the middle of every 10 or 12 Inches of the depth, and these may be thus found.

Take (with the long Rule and Cane, or some other convenient Instrument) a Diameter in the middle of every 10 Inches of the Tuns depth, and suppose in this Example the first (at 5 Inches from the top) be 95.5, the second (at 15 Inches from the top) 100.5, and the third 105.5, these I set down as in the following Table.

This done, we may consider the Tun either as Round or Square, and first to Inch it as a Round Tun.

Set the Gage-Point (*a g*) to 1; then against the several Diameters already found, you have their respective Area's: So the first Area will be 25.40, the second 28.31, the third 31.00, these I set down in the Table against their Diameters, and having added them together, the Sum is 84.53, this Multiplied by 10 (or which is all one, remove the Prick one place towards the Right hand) the Product is 845.3, the whole Content of the Tun.

Note,

Note, Here the Diameters being taken in the middle of every 10 Inches, there will be no uſe of Multiplication or Diviſion in Inching the Tun, as there will be when the Diameters are taken in every Foot or half Foot, for here the Area's of the ſeveral Diameters do not only ſhew the mean Inch in the middle of every 10 Inches, but alſo the whole Content of thoſe 10 Inches, if you remove the Prick but one place more towards the Right hand. Thus the mean Inch of the laſt 10 Inches is 31.00, but if the Prick be removed as aboveſaid, it is 310, which is the Content of the Tun at 10 Inches deep, I therefore reduce the ſeveral Contents in the third Column into Barrels, Firkins and Gallons of Beer meaſure, and ſet them down in the fourth Column, theſe added together are 23 Barrels, 1 Firkin, 8 Gallons and 3 tenths of a Gallon, which agrees with the Sum of the Numbers in the third Column.

Parts of the Tuns depth.	*Mean Diameters in the middle of every 10 Inch. of the Tuns dep.*	*Mean Area's or Contents of every 10 Inch. of the Tuns dep.*	*Content of every 10 Inches Beer Meaſure.* B. F. G.
10	95.5	25.40	7 0 2.0
10	100.5	28.13	7 3 2.3
10	105.5	31.00	8 2 4.0
30		84.53	23 1 8.3

By theſe Numbers thus found it will be eaſie to make a Table which ſhall ſhew the quantity of Liquor contain'd in the Tun at any depth, for which obſerve the following directions.

1. Set

1. Set down the whole Content of the Tun in Barrels, Firkins, Gallons, and parts, in their respective Columns, and at the top of the first Column against the Content set 0, and under it the Tuns depth in Inches 1, 2, 3, 4, 5, &c. this done take the first mean Inches, *viz.* 25.4 and reduce it into Barrels, Firkins, and Gallons, Beer Measure, it makes 0 : 2 : 7 : 4 : set this upon a scroll of Paper, and Subtract it continually from the whole Content 'till you come at the 10th. Inch of the Tuns depth, which done, if you mistake not, the remainder there will be 16 : 1 : 6 : 3.

2. Take the second mean Inch, *viz.* 28, 13, this being reduced as the former is 0 : 3 : 1.13, Subtract this continually from the last remainder, 'till you come at 20 Inches deep, and there your remainder will be 8, 2, 4.

3. Take the third and last mean Inch and reduce and Subtract it as before, and if your Work be right there will be no remainder at the last Inch or bottom of the Tun; now the Table being made you may cast away the Fractions as useless. The use of this Table is obvious to the meanest Capacity, without any further explanation: For if you come to this Tun and

In.	B.	F.	G.
0	23	1	8.30
1	22	3	0.9
2	22	0	2.5
3	21	1	4.1
4	20	2	5.7
5	19	3	7.3
6	19	0	8.9
7	18	2	1.5
8	17	3	3.1
9	17	0	4.7
10	16	1	6.3
11	15	2	5.17
12	14	3	4.04
13	14	0	2.91
14	13	1	1.78
15	12	2	0.65
16	11	2	8.52
17	10	3	7.39
18	10	0	6.26
19	9	1	5.13
20	8	2	4.00
21	7	3	0
22	6	3	5
23	6	0	1
24	5	0	6
25	4	1	2
26	3	1	7
27	2	2	3
28	1	2	8
29	0	3	4
30	0	0	0

find

find 2 Inches of the depth dry, the quantity of Liquor then in the Tun is 22 B. 0 F. 2 G. If 7 Inches be dry, there remain in the Tun but 18, 2, 1, *&c.* Thus for Round Tuns.

Now suppose the Tun last mentioned were Square, and the sides at bottom and top equal to the Diameters of the last, and the depth the same, the Diameters or sides in the middle of every 10 Inches of the Tuns depth will be the same as in the last, by which the several mean Inches, and also the whole Contents of every 10 Inches are found to be as in this Table.

Parts Tuns depth.	*Sides of the Tun in the middle of every 10 Inches.*	*Mean Area's or Content of every 10 Inches.*	*Content of every 10 Inches Beer Measure.* B. F. G.
10	95.5	32.34	8 2 8.4
10	100.5	35.81	9 2 6.1
10	105.5	39.46	10 3 7.6
30		107.61	29 2 4.1

For, Set 282 upon B, to 95.5 the first Diameter upon A, then against 95.5 upon B is 32.34 the first mean Inch, and after the same manner find the second and third, which being reduced as in the last Example, I find the whole Content to be 1076.1 Ale Gallons (or which is the same thing) 29 B. 2 F. 4.1 G. this being done you may proceed to Inch this Tun by the same directions which were given in the last Example.

Problem

Problem III.

To Inch a Tun whose Bases are unlike, viz. *the lower Circular, and the upper Elliptical, suppose the Diameter of the Circular Base be* 100 *Inches, the Axis (or longest Diameter) of the Elliptical Base* 100, *and the shortest* 90, *and the Tuns depth* 20 *Inches.*

TO Inch this Tun, I take (with the long Rule) the cross Diameters in the middle of every 4 Inches of the Tuns depth: Thus at 2 Inches from the top I find the longest Diameter 100, and the shortest 90, in the middle of the next four Inches (which is 6 Inches from the top) the longest Diameter is 100, and the shortest 91, and thus I find the rest which are as in this Table is express'd: Now having these Diameters, the Area or mean Inch in the middle of every 4 Inches of the Tuns depth may be thus found.

Set 359.05 upon B, to the longest Diameter, *viz.* 100 upon A: then against the shorter Diameter upon B, is the Area in Ale Gallons; thus against 91 is 25.34, the Area in the first 4 Inches, against 93 is 25.88 the second Area. In like manner against 95, you have 26.47 the third mean Inch: And so of the others as in the Table.

Depth.

Depth.	Axis.	Shorter Diam	Mean Inch in Ale Gallons.	Content of every 4 Inches of the Tuns Depth.		
4	100	91	25.34	2	3	2.36
4	100	93	25.88	2	3	4.52
4	100	95	26.47	2	3	6.88
4	100	97	27.02	3	0	0.08
4	100	99	27.57	3	0	2.28
20			132.28	14	2	7.12

Now if the mean Inches be ſeverally Multiplied by 4, and theſe Products reduced into Barrels, Firkins, and Gallons, Beer meaſure, they will give the Numbers in the laſt Column, which added together are 14 Barrels, 2 Firkins, 7 Gallons, and .12 parts of a Gallon.

Set this down for the firſt Number of your Table, and then reduce the firſt mean Inch into Barrels, Firkins, and Gallons, and Subtract it 4 times from the whole Content and remainders; do the like with the other mean Inches, and ſo complete the Table as in the laſt Example.

Problem IV.

To find the Drip or Fall of a Tun, and to make allowance for the ſame in Inching the Tun.

As for Inſtance.

ADmit the Tun laſt mention'd were ſo plac'd, that when the bottom is but juſt cover'd on one

one ſide, the Liquor is 4 Inches deep on the ſide oppoſite; How much muſt be allow'd for the fall of this Tun.

By the operation aforegoing the Area of the mean Diameter in the middle of the 4 laſt Inches (that is at 2 Inches from the bottom) was found to be 27.57, this Multiplied by 4, is 110.28 Ale-Gallons, the half of this is 55.14, and ſo much may be allow'd for the fall of the Tun, I therefore reduce this 55.14 into Barrels, Firkins, and Gallons Beer Meaſure, it gives 1 : 2 : 1.14: this taken from 14 : 2 : 7.12 the Content of the Tun when it ſtands level, there remains 13 Barrels, 0 Firkins, 3 Gallons, and .98 parts: and ſo much will the Tun contain, when the fall is four Inches, I therefore ſet this at the top of the Table, then reducing the firſt mean Inch into Barrels, Firkins and Gallons, I Subtract it four times from the whole Content, the remainder is 10 : 1 : 3.62: Do the like with ſecond, third and fourth mean Inches, and you find the remainder at 16 Inches dry to be 1 : 2 : 1.14: and ſo much it takes to cover the bottom as was before obſerved.

0	13	0	5.98
1	12	1	7.64
2	11	3	0.30
3	11	0	1.96
4	10	1	3.62
5	9	2	4.74
6	8	3	5.86
7	8	0	6.98
8	7	1	8.10
9	6	2	8.63
10	6	0	0.16
11	5	1	0.69
12	4	2	1.22
13	3	3	1.20
14	3	0	1.18
15	2	1	1.16
16	1	2	1.14

As for ſuch Tuns as are ſuppoſed to be the Fruſtums of Spheroids, Parabolick Conoids, Hyparabolick Conoids, or Parabolick Spindles, I ſhall give Rules for Gaging and Inching of theſe in the Appendix, where I ſhall alſo endeavour to explain the nature of thoſe Figures whence they are derived.

Problem

Problem V. Figure 20.

To Gage a Copper and make allowance for the Crown; let a, b, c, d, e, a *represent a Copper whose Content is required.*

1. TO find the several Diameters and Depth of the Copper, together with the Altitude of the Crown.

Take a small Cord or Thread, make one end fast at *a*, and extend the other to the opposite side of the Copper at *e*, where make it fast, or cause some person to hold it very strait; this done, set one end of the Instrument in the bottom of the Copper at *b*, and move it to and fro till you find the nearest distance to the Thread as at *o*, this distance *b o*, is the depth of the Copper, suppose it be 44 Inches.

In like manner set the end of the Rule upon the top of the Crown at *c*, and take the nearest distance to the Thread as *c v*, suppose it be 36 Inches, this subtracted from *b o* 44, the remainder 8, is the Altitude of the Crown.

To find *b d* the Diameter of the bottom of the Crown:

Measure *a e*, the Diameter at the top, admit it be 115 Inches, then hold a Thread so as a Plummet at the end thereof may hang just over *b*; by this means you may find the distance *a o*, and on the other side *e s*, suppose each be 14.5, add these together and Subtract their Sum (*viz.* 29) from *a e*, 115 the remainder 86 is equal to *b d*, the Diameter at the bottom of the Crown, the Diameter which touches the top of the Crown may be found by the Instrument, suppose it be 91.4 Inches.

Now to find the Content of the Copper from the Crown upwards, (that is, the part *a b c i e*) the depth *v c* being 36 Inches, you may take a Diameter in the middle of every 6 Inches betwixt *v* and *c*, which ſuppoſe to be as in the ſecond Column of the following Table, the Numbers in the third Column are the reſpective Area in Ale Gallons, found by the Gage-Point as before; the fourth Column ſhews the Content of every 6 Inches in Barrels, Firkins and Gallons, Beer meaſure: Theſe added together are 29 Barrels, 3 Firkins, 2.3 Gallons.

And ſo much will the Copper contain after the Crown is cover'd.

Now if the Crown be taken for the Fruſtum of a Sphere, the Content (by the third Problem of Section V.) will be found to be 83.3 Gallons.

But the Content may be more readily found and very near the truth: thus,

The Diameter *b d*, was found to be 86, the Area to this Diameter is 20.6, this Multiplied by $\frac{1}{2}$ the Crowns Altitude, *viz.* 4, gives 82.4, the Content of the Crown, this Subtracted from the Content of the part or portion *b c i d n b*, the remainder is the quantity of Liquor that it will require to cover the Crown; the Content of the part *b c i d n b* may be thus found, *b i* is 91.4, and *b d* is 86, theſe added together are 177.4, the half 88.7 may be taken for a mean Diameter, the Area to this Diameter is 21.9, this Multiplied by 8, the Altitude, gives 175.2 the Content.

Now Subtract 82.4 (the Content of the Crown) from 175.2, the remainder is 92.8, or 2 : 2 : 28, this added to the Content firſt found, *viz.* 29 : 3 : 2.5, gives 32 : 1 : 5 the Content of the whole Copper.

Parts

Parts of the depth.	*Diameter*	*Mean Inches.*	*Content of every 6 Inches in* B.	F.	G.
6	113.1	35.62	5	3	6.7
6	109.2	33.21	5	2	1.2
6	105.28	30.86	5	0	5.1
6	101.36	28.61	4	3	0.6
9	97.33	26.37	4	1	5.2
6	93.4	24.29	4	0	1.7
36	Sum		29	3	2.5
To cover the Crown			2	2	2.8
Content Copp.			32	1	5.3

The Content being thus found you may proceed to Inch the Copper by the ſame directions which were given for Inching of Tuns in the laſt Problem.

Problem VI.

The Diameter of any Round or Side of any Square Maſh-Tun, being given, to find how many Gallons of Mault it will contain at one Inch deep.

THE Corn Gallon (which is the meaſure we here account by) is ſuppoſed to contain 272.25 Cubick Inches.

But Mr. *Dary* in his Complete Gager ſaith, that an indifferent ſort of Mault, and of an indifferent Grinding, when three Worts ſhall have paſſed through it, is more compact by about a ſixth part; he there-

fore concludes that 227 may be a proper Divisor for Square Mash-Tuns, and 288 for Round ones.

1. *For Round Mash-Tuns.*

The Diameter of that Circle whose Area is 227 is 17, and this may be called the Gage-Point for Round Mash-Tuns, and to find the Content of any such:

Set 17 upon D, to 1 upon C; then against any Diameter upon D, you have the Content in Gallons at one Inch deep: thus if the Diameter were 113, the Area or Content at one Inch deep would be 44.3 that is, 5 Bushels and 4 Gallons.

2. *For Square Mash-Tuns.*

If a Mash-Tun be Square or Oblong, the Content at one Inch deep may be thus found: Say,

As 227 is to the Length,
So is the Breadth to the Content.

Example.

Suppose the length 112 Inches, and the breadth 80, how many Gallons will this Tun contain at one Inch deep? Answer 39.4: For,

Set 227 upon A, to 80 upon B; then against 112 upon A, is 39.4 the Content upon B.

SECT.

SECT. IX.

Of Cask Gaging.

BEfore we can find the Content of a Cask, we muſt know the Dimenſions of it, *viz.*

The { Bung Diameter, / Head Diameter, / Length, } in Inches.

And when theſe are known, the form or ſhape of the Cask muſt be conſidered, for although the Diameters (aboveſaid) and the length of one Cask may be equal to thoſe of another, yet one of theſe Casks may contain ſeveral Gallons more than the other, and therefore the Content of all Casks cannot be found by one and the ſame Rule.

Now moſt Writers in Treating on this Subject have taken it for granted, that every common Cask (as Butts, Pipes, Hogſheads, Barrels and ſuch like) is in the form of one of the Solids, *viz.*

1. The middle Fruſtum of a Spheroid.
2. The middle Fruſtum of a Parabolical Spindle.
3. The middle Fruſtum of two Parabolical Conoids, abutting upon one common Baſe, or
4. The middle Fruſtum of two Cones abutting upon one common Baſe.

They have not (as I remember) told us what they mean by all thoſe terms, but have laid down Rules for finding the Content of theſe Solids, and by theſe

Rules they ſuppoſe the Content of any Cask may be alſo found.

As for inſtance,

If the Dimenſions of a Cask be

Bung } Diameter — } 34
Head } } 24 } Inches.
Length — — — 48 }

By the Rules given by Mr. *Dary*, and others, the Content of this Cask is taken for

			Ale Gall.
The middle Fruſtum of	2	Spheroid — — —	128.71
		Parabolical Spindle	127.64
	2	Parabolic. Conoids	115.77
		Cones — — —	113.55

Dr. *Newton* in his Coſmography page 81. gives a Rule by which the Content of this Fruſtum of a Parabolical Spindle will be but 118.37, and according to others it ſhould be but 110 87 Gallons.

But they all agree in giving true Rules for the firſt and laſt of theſe Solids, *viz.*

The middle Fruſtum of { a Spheroid, two Cones.

And theſe may both be repreſented by Figure 21.

A Cask which is the middle Fruſtum of a Spheroid is ſhewn by the outermoſt Line *d,o,a,o,d,f,c,o,b,o,c,e,d,* the two ſtraight Lines *d, e, c,* and *d, f, c,* are the Heads of the Cask, and the Curved Lines *d,o,a,o,d,* and *c,o,o,c,* the Staves.

If

If a Cask (can) be in the form of the middle Fruſtum of two Cones, it may be repreſented by the ſtraight Lines *d, u, a, u, d,* and *c, u, b, u, c,* the Heads *d, e, c,* and *d, f, c,* the Bung Diameter *a, b,* and the Length *e, f,* are all the ſame in both theſe Casks, and yet 'tis evident by the Figure it ſelf, that one will contain more than the other, and by the Example above Cited the Difference is 15.16 Gallons, for the Spheroid Contains 128.71 Gallons, and the Fruſtum of two Cones but 113.55.

There are very few (if any) Casks whoſe Diameters and Length are equal to thoſe above named, that will contain more than 128.71 Gallons (the Content of the Spheroid) nor leſs than 113.55 (the Content of the Fruſtum of two Cones) ſo that all the varieties that can happen will fall betwixt theſe two, and the Staves of any Cask differing from theſe will lie between the Lines *d, o, a,* and *d, u, a,* in Figure 21.

Now for the other two Solids, *viz.* the middle Fruſtum of a Parabolical Spindle, and the middle Fruſtum of two Parabolical Conoids; according to the Authors above cited, the Content of the firſt is 127.64 Gallons, and the ſecond 115.77.

Whether the Rules they have given for finding theſe Contents be true or falſe, I ſhall not here examine.

But ſuppoſing they are true, yet they do not anſwer to the various caſes that happen in common Practice, for in the Example above, the Content of a Cask taken, as the middle Fruſtum of a Parabolical Spindle is but 1.07 leſs than the Content of the Fruſtum of a Spheroid, and the Content of the Fruſtum of two Parabolical Conoids is but 2.22 more than the Fruſtum of two Cones.

Now I presume that no Man will pretend to distinguish which Notion a Cask must be taken in, whether as the Frustum of a Spheroid, or as the Frustum of a Parabolical Spindle, when the Contents differ but one Gallon in 128.

Moreover, here is almost 12 Gallons difference betwixt the Content of the Frustum of a Parabolical Spindle, and the Frustum of two Parabolical Conoids: So that I may modestly affirm that there are many Casks whose Content cannot be found by any of these Rules; for it must be granted that a Cask whose Diameters and Length are equal to those in this Example, may contain some certain quantity betwixt 127 Gallons, and 115.

As suppose 123, I say if this Cask were to be Gaged by any of the Rules above cited, they will give the Content at least 4 Gallons too much, or 7 Gallons too little.

This premised, I come now to lay down some short Rules by which the Content of any Cask may be found very near the truth.

Problem

Problem I. Figure 21.

There is a Cask supposed to be the middle Frustum of a Spheroid, intercepted between two plains Parallel, cutting the Axis at Right Angles, let a b *the Diameter at the Bung be* 34 *Inches,* d c *the Diameter at the Head* 24, *and the Length* c f 48 *Inches; How many Ale or Wine Gallons will this Cask contain?*

Rule.

TO the Sum and half Sum of the Squares of the Bung and Head Diameters, add half the difference of the said Squares, the Sum of these Multiplied by the Length, and this Product divided

By { 1077 / 882 } { Quotes the Content in } { Ale / Wine } Gallons.

Example.

Square of *a b* 34 is	1156
Square of *d c* 24 is	576
Sum is	1732
Half Sum	866
Half the difference of the Squares is	290
Sum is	2888

Which Multiplied by 48 (the Length of the Cask) gives 138624, this Divided by 1077 Quotes 128.71 the Content in Ale Gallons.

Or Divided by 882 the Quotient will be 157.17 the Content in Wine Gallons.

Note, the Common Rule for Gaging the Fruſtum of a Spheroid, (by taking twice the Square of the Bung, and the Square of the Head, *&c.*) is ſomewhat a ſhorter way than this: but this Rule being more agreeable to the Work in the following Problem, I have thought fit to inſert it here: the Reader may uſe which he pleaſe.

Problem II.

Suppoſe a Cask (of the ſame Diameters, and Length with the former) ſhall have its Staves betwixt the Bung and Head leſs Curved, ſo as to leave out about a third part of the ſpace between the Lines d o a, *(which repreſents the Spheroid) and* d u a, *as in Figure* 22. *How many Gallons will this Cask Contain?*

Rule.

TO the Sum of the Squares of the Bung and Head Diameters, add three tenth parts of the Difference of the ſaid Squares, the Sum of theſe Multiplied by the Length, and the laſt Product Divided by 1077, or 882 Quotes the Content in Ale or Wine Gallons reſpectively.

Example.

Example. *The Dimensions as above.*

Sum of the Squares is	1732
Half Sum is	866
The Difference of the Squares Multiplied by .3 is	174
Sum of these is	2772

This Multiplied by 48, gives 133056, and this Divided by 1077 quotes 123.54 the Content in Ale Gallons, or by 882, the Quotient will be 150.85 the Content in Wine Gallons.

Problem III. Figure 19.

There is a Cask whose Staves, between the Bung and Head, have but little Curveture, lying nearer to the straight Line a u d, *than to the Curved Line* a o d, *what is the Content of this Cask in Ale or Wine Gallons?*

Rule.

TO the Sum and half Sum of the Squares of the Bung and Head Diameters, add one tenth of the Difference of the said Squares, the Sum of these Multiplied by the Length, and this Product Divided as above directed, quotes the Content in Ale or Wine Gallons respectively.

Example.

Example. (*Dimenſions as above.*)

Sum of the Square is	1732
Half Sum is	866
One tenth of the Difference is	58
Sum of theſe is	2656

Which Multiplied by 48, gives 127488, this Divided by 1077 the Quotient is 118.37, the Content in Ale Gallons; or by 882 quotes 144.54, the Content in Wine Gallons.

Problem IV. Figure 21.

To find the Content of a Cask in the form of the middle Fruſtum of two Cones abutting upon one common Baſe (if any ſuch can be) it may be repreſented by the ſtraight Lines d u a, a u d.

Rule:

FRom the Sum and half Sum of the Squares of the Bung and Head Diameters, Subtract half the Square of the Diameters themſelves, Multiply the remainder by the Length, and Divide this Product as in the laſt, the Quotient will be the Content in Ale or Wine Gallons.

Example.

Example. (*Dimensions as above.*)

Sum of the Square is	1732
Half Sum is	866
Sum of these is	2598
Difference of the Diameter is 10, this Squared is 100, the half is	50
The Remainder is	2548

This Multiplied by 48, gives 122304, and this Divided by 1077 quotes 113.55, the Content in Ale Gallons; or by 882 the Quotient will be 138.66, the Content in Wine Gallons.

Thus it appears by the four Problems aforegoing, if the Dimensions of a Cask

Be	Bung Diameter	34	Inches.
	Head Diameter	24	
	Length	48	

The Content in each variety is as followeth.

		Ale Gall.	Diff.
By Problem	I.	128.71	5.17
	II.	123.54	5.17
	III.	118.37	4.82
	IV.	113.55	

And here it may be observed that the difference between the Content of the first and second, is equal to the difference between the second and third, and the difference between the third and fourth, is not above three Pints less than the other: so that by these

theſe Rules there is not ſo much room for error, as is by the common Method which ſeveral Authors have preſcribed, for by their Rules the Content of this Cask in the ſecond Notion would be but one Gallon leſs than in the firſt, and the Content in the third Notion but little above two Gallons more than in the fourth, and ſo the difference between the Second and third would be about 12 Gallons.

It may yet be ſuppoſed (and muſt be granted) that a Cask of theſe Diameters and Length, may contain ſome certain quantity between the Contents here found, as ſuppoſe 126, 121 or 116 Gallons.

In ſuch Caſes you may proceed by the Rule already given, adding 1, 2, 3, 4, or 5 tenth parts of the Difference of the Squares, as your judgment and the ſhape of the Cask ſhall direct: and the like caution muſt be uſed in finding the mean Diameter, if you work by the Inſtrument. But when the difference of the Diameters of any Cask is but 4 or 5 Inches, the four Rules aforegoing (or indeed the three firſt of them) will anſwer all the varieties that can happen: as ſuppoſe the Dimenſions of a Cask be

Bung Diameter —— 28
Head Diameter —— 24 } Inches.
Length —— 40

The Content in Wine Gallons will be found to be as followeth,

			Diff.
By Problem	I.	97.22	1.88
	II.	95.34	1.88
	III.	93.46	1.31
	IV.	92.15	

I have

I have insisted the longer upon this business that I might prevent mistakes, for we cannot be too exact, where a small errour must needs be prejudicial either to the King, or to the Merchant, especially when the Duty upon Brandy is great.

Problem V.

The Dimension of any Cask being given, to find the Content by the Instrument.

TO do this we must first find a mean Diameter, that is, such a Diameter as shall reduce the Cask to a Cylinder, for which purpose I have Calculated these three Tables, for the three first varieties of Casks abovementioned.

A Table for Equating the Diameters of the middle Frustum of a Spheroid.

1	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.67	0.73	0.80	0.87	0.94	1.01	1.08	1.15	1.22	1.29
2	1.36	1.43	1.50	1.57	1.64	1.71	1.78	1.85	1.92	1.99
3	2.06	2.13	2.20	2.27	2.34	2.41	2.48	2.55	2.62	2.69
4	2.76	2.83	2.90	2.97	3.04	3.11	3.18	3.25	3.32	3.39
5	3.46	3.54	3.61	3.68	3.75	3.82	3.89	3.96	4.03	4.10
6	4.17	4.24	4.31	4.39	4.46	4.53	4.60	4.67	4.74	4.81
7	4.88	4.95	5.03	5.10	5.17	5.25	5.32	5.39	5.46	5.53
8	5.60	5.68	5.75	5.82	5.90	5.97	6.04	6.11	6.18	6.25
9	6.32	6.40	6.47	6.55	6.62	6.70	6.77	6.84	6.91	6.98
10	7.05	7.13	7.20	7.28	7.35	7.42	7.49	7.56	7.64	7.71

A Table for the Second Variety of Casks.

1	0	1	.1	1	.2	1	.3	1	.4	1	.5	1	.6	1	.7	1	.8	1	.9
1	0.62		0.68		0.74		0.81		0.87		0.93		0.99		1.06		1.12		1.18
2	1.24		1.30		1.36		1.43		1.49		1.55		1.62		1.68		1.74		1.80
3	1.86		1.93		1.99		2.05		2.12		2.18		2.24		2.30		2.37		2.43
4	2.49		2.55		2.62		2.68		2.75		2.81		2.88		2.94		3.01		3.07
5	3.13		3.20		3.26		3.33		3.39		3.46		3.52		3.59		3.65		3.72
6	3.78		3.85		3.91		3.98		4.04		4.11		4.18		4.24		4.31		4.37
7	4.44		4.50		4.57		4.63		4.70		4.76		4.83		4.90		4.96		5.03
8	5.10		5.16		5.23		5.30		5.36		5.43		5.49		5.56		5.63		5.70
9	5.77		5.84		5.91		5.97		6.03		6.10		6.17		6.24		6.30		6.37
10	6.44		6.51		6.57		6.64		6.71		6.77		6.83		6.90		6.96		7.02

A Table for the Third Variety of Casks.

1	0	1	.1	1	.2	1	.3	1	.4	1	.5	1	.6	1	.7	1	8.	1	.9
1	0.55		0.60		0.65		0.70		0.76		0.81		0.87		0.93		0.99		1.05
2	1.11		1.17		1.22		1.28		1.33		1.39		1.44		1.50		1.55		1.61
3	1.67		1.73		1.78		1.84		1.90		1.95		2.01		2.06		2.12		2.18
4	2.24		2.30		2.35		2.41		2.46		2.52		2.57		2.63		2.69		2.75
5	2.81		2.87		2.93		2.98		3.04		3.10		3.16		3.21		3.27		3.33
6	3.39		3.45		3.51		3.56		3.62		3.68		3.73		3.80		3.85		3.91
7	3.97		4.02		4.08		4.14		4.20		4.26		4.32		4.38		4.44		4.50
8	4.56		4.62		4.68		4.74		4.80		4.86		4.92		4.98		5.04		5.10
9	5.16		5.22		5.28		5.35		5.41		5.47		5.53		5.59		5.65		5.71
10	5.77		5.83		5.89		5.96		6.02		6.08		6.14		6.20		6.26		6.32

Now

Now by these Tables the mean Diameter of any Cask is thus found.

Subtract the Head Diameter from the Bung Diameter, then seek the difference in the Table, taking the whole Inches in the first Column, and the length (if any be) at the Head, in the Angle of meeting you have a Number, which being added to the Head Diameter, gives the mean Diameter sought.

Example.

Let the Dimensions be as in the first Problem of this Section, the difference of the Diameters is 10. In the Table for a Spheroid, against 10 on the side, and under 0, I find 7.05, this added to the Head Diameter (*viz.* 24) the Sum is 31.05, the mean Diameter sought.

This being obtained, the Content of any Cask may be found by the Instrument at one Operation, for which the Proportion is:

As the Gage-Point is to the Casks length,
So is the mean Diameter to the Content.

Example.

The length 48, and the mean Diameter 31.05, Inches.

Set the Gage-Point for Ale Gallons to 48 upon C, then against 31.05 upon D, is 128.71 the Content in Ale Gallons, which is the same as was found by the first Problem of this Section.

Note, The three Tables above are expressed in three Scales, and placed upon one side of the Rule against the Line of Inches, by which the mean Diameter of any Cask may be readily found.

To

To find the Ullage of a Cask.

Problem VI.

Let Figure 21. repreſent a Cask poſited with its Axis e f, *Parallel to the Horizon, in part empty the Surface of the Liquor cutting the Bung Diameter in* S, *ſuppoſe the whole Content* 128.71 *Gallons, the Bung Diameter* a b 34 *Inches, the wet part* s b 26, *the dry part* S a 8; *How many Gallons are contained in the Cask? and how many will it require to fill it full.*

FOR reſolving this, and ſuch like Queſtions, there is a Line of Segments on the Rule, with a Line of Numbers to ſlide againſt it, and upon theſe Lines the proportion is:

1. *As* the Bung Diameter upon the Numbers, is to 100 upon the Segments,

So is the wet or dry parts on the Numbers, to a fourth Number on the Segments.

2. *As* 100 upon B, is to the Casks whole Content upon A,

So is the fourth Number laſt found to the Anſwer.

Example.

1. Set 34 the Bung Diameter on the Numbers, to 100 on the Segments, then againſt 8 the dry Inches on the Numbers is 16 on the Segments.

2. Set

2. Set 100 upon B, to 128.71 the Casks whole Content upon A, then againſt 16 upon B, you have 20.6 upon A, and ſo many Gallons it will take to fill up the Cask, this Subtracted from 128.71, the whole Content, there reſts 108.11 the quantity of Liquor in the Cask; which may otherwiſe be found thus, having ſet 34 the Bung Diameter to 100 on the Segments, againſt 26 the wet Inches on the Numbers, you have 84 on the Segments.

Then,

As 100 upon B, is to 128.71 upon A,
So is 84 upon B, to 108.11 upon A:
Which being the ſame as before is a proof of the Work.

Note, The Line of Segments on this Rule is made for Spheroid, and is therefore more exact for all Bulging Casks than the Tables of Segments, which are proper only for a Cylinder.

The following Tables ſhew the Ullage of two ſeveral Casks to every Quart or half Gallons, they were made by drawing out the Liquor by an exact Wine Quart, and the dry Inches taken by an Inſtrument proper for that purpoſe.

A Table shewing the Ullage of a Cask to every Quart or half Gallon, the Length of the Cask is 37.3 Inches, the Bung Diameter 29.5, the Head Diameter 25, and the Content 100 Wine Gallons.

Dry Inches.	Gallons and Quarts.		Dry Inches.	Gallons and Quarts.		Dry Inches	Gallons and Quarts.	
0.71		1	6.19	13	2	10.98	32	2
1.10		2	6.33	14	0	11.09	33	0
1.40		3	6.48	14	2	11.20	33	2
1.62	1	0	6.62	15	0	11.31	34	0
1.82	1	1	6.76	15	2	11.42	34	2
2.00	1	2	6.90	16	0	11.53	35	0
2.16	1	3	7.04	16	2	11.64	35	2
2.30	2	0	7.18	17	0	11.75	36	0
2.44	2	1	7.32	17	2	11.86	36	2
2.57	2	2	7.46	18	0	11.97	37	0
2.69	2	3	7.60	18	2	12.08	37	2
2.80	3	0	7.73	19	0	12.19	38	0
2.91	3	1	7.87	19	2	12.30	38	2
3.01	3	2	8.00	20	0	12.41	39	0
3.11	3	3	8.14	20	2	12.52	39	2
3.20	4	0	8.26	21	0	12.63	40	0
3.30	4	1	8.39	21	2	12.74	40	2
3.39	4	2	8.52	22	0	12.85	41	0
3.48	4	3	8.65	22	2	12.96	41	2
3.57	5	0	8.77	23	0	13.07	42	0
3.65	5	1	8.90	23	2	13.18	42	2
3.73	5	2	9.02	24	0	13.28	43	0
3.81	5	3	9.14	24	2	13.39	43	2
3.89	6	0	9.26	25	0	13.49	44	0
4.05	6	2	9.38	25	2	13.60	44	2
4.21	7	0	9.50	26	0	13.70	45	0
4.37	7	2	9.62	26	2	13.81	45	2
4.53	8	0	9.73	27	0	13.92	46	0
4.69	8	2	9.85	27	2	14.02	46	2
4.84	9	0	9.96	28	0	14.12	47	0
4.99	9	2	10.08	28	2	14.23	47	2
5.15	10	0	10.19	29	0	14.33	48	0
5.30	10	2	10.31	29	2	14.44	48	2
5.45	11	0	10.42	30	0	14 54	49	0
5.60	11	2	10.54	30	2	14.65	49	2
5.75	12	0	10.65	31	0	14.75	50	0
5.90	12	2	10.76	31	2			
6 04	13	0	10.87	32	0			

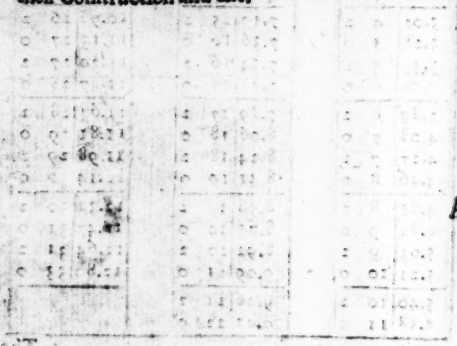
A Table ſhewing the Ullage of a Cask to every half Gallon, the Length of the Cask is 30.8 Inches the Bung Diameter 25.6, the Head Diameter 22.6, and the Content 64 Wine Gallons.

Dry Inch.	Gall. and Quar.	Dry Inch.	Gall. and Quar.	Dry Inch.	Gall. and Quar.
.80	2	5.76	11 2	9.60	22 2
1.25	1 0	5.94	12 0	9.77	23 0
1.60	1 2	6.12	12 2	9.94	23 2
1.91	2 0	6.13	13 0	10.11	24 0
2.17	2 2	6.48	13 2	10.28	24 2
2.40	3 0	6.66	14 0	10.45	25 0
2.63	3 2	6.84	14 2	10.62	25 2
2.85	4 0	7.01	15 0	10.79	26 0
3.07	4 2	7.19	15 2	10.96	26 2
3.28	5 0	7.36	16 0	11.13	27 0
3.49	5 2	7.54	16 2	11.30	27 2
3.69	6 0	7.71	17 0	11.47	28 0
3.89	6 2	7.89	17 2	11.64	28 2
4.08	7 0	8.06	18 0	11.81	29 0
4.27	7 2	8.24	18 2	11.98	29 2
4.46	8 0	8.41	19 0	12.14	30 0
4.65	8 2	8.58	19 2	12.31	30 2
4.84	9 0	8.75	20 0	12.47	31 0
5.03	9 2	8.92	20 2	12.64	31 2
5.21	10 0	9.09	21 0	12.80	33 0
5.40	10 2	9.26	21 2		
5.58	11 0	9.43	22 0		

The

The Casks by which these Tables were made, were a Brandy Puntion, and a Brandy Hogshead, both from *Bourdeaux*: the Tables will serve indifferently for all such Casks, whose Diameters Length, and Content are the same with these, and are so plain that there needs no Example to shew their use; for against the dry Inches you have the Gallons and Quarts that the Cask wants of being full, and if the Bung Diameter be more than half dry, enter the Table with the wet Inches instead of the dry, it shews the quantity of Liquor in the Cask: These Tables may by proportion be converted into Lines, and put upon a Rule, as they have sometimes been by Mr. *Isaac Carver*, who is a very accurate Man at all such Operations.

By what hath been already said 'tis easie (I presume) to find the Ullage of a Cask whose Axis is posited Parallel to the Horizon: but lest this Tract should be thought imperfect, I shall here infert a Table of the Segments of a Circle, and also shew their Construction and Use.

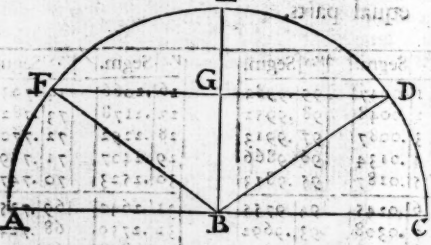


A Table of the Segments of a Circle whose Area is Unity, the Diameter (*viz.* 1.128378) being divided into 100 equal parts.

Vs	Segm.	Vs	Segm.	Vs	Segm.	Vs	Segm.
1	.0017	99	.9983	26	.2066	74	.7934
2	.0048	98	.9952	27	.2178	73	.7822
3	.0087	97	.9913	28	.2292	72	.7708
4	.0134	96	.9866	29	.2407	71	.7593
5	.0187	95	.9813	30	.2523	70	.7477
6	.0245	94	.9755	31	.2640	69	.7350
7	.0308	93	.9692	32	.2759	68	.7241
8	.0375	92	.9625	33	.2878	67	.7122
9	.0446	91	.9554	34	.2998	66	.7002
10	.0520	90	.9480	35	.3119	65	.6881
11	.0598	89	.9402	36	.3241	64	.6759
12	.0686	88	.9320	37	.3364	63	.6436
13	.0764	87	.9236	39	.3487	62	.6513
14	.0851	86	.9149	38	.3611	61	.6389
15	.0941	85	.9059	40	.3735	60	.6265
16	.1033	84	.8967	41	.3860	59	.6140
17	.1127	83	.8873	42	.3986	58	.6014
18	.1224	82	.8776	43	.4112	57	.5888
19	.1323	81	.8677	44	.4238	56	.5762
20	.1424	80	.8576	45	.4364	55	.5636
21	.1526	79	.8474	46	.4491	54	.5509
22	.1631	78	.8369	47	.4618	53	.5382
23	.1737	77	.8263	48	.4745	52	.5255
24	.1845	76	.8155	49	.4873	51	.5127
25	.1955	75	.8045	50	.5000	50	.5000

The

The Construction of the Table of Segments of a Circle.



In the Figure here before us, all that part or portion comprehended within the Right Lines D B and B F, and the Circular Line F E D is called the Sector of a Circle, F G D is the Cord of the Segment, G E is the Verſed Sine, and G B the Complement thereof, A B C is the Diameter of the Circle, which here we ſuppoſe to be 2.

Now if the Diameter of a Circle be 2, the length of the Circumference is double the Area of the Circle, for the Circumference is 6.2831852, and the Area 3.1415926; hence it evidently follows that the length of half the Arch of any Sector of a Circle, is equal to the Area of that Sector, and (as ſhall be proved anon) the Natural Sine of the ſaid half Arch is equal to the Area of the Triangle in that Sector.

Now the Area of the Triangle taken from the Area of the Sector leaves the Area of the Segment.

Example.

Example.

Let it be required to find the Area of the Segment F E D G, whose verſed Sine G E is .4.

1. To find the Area of the Sector.

In order to which we muſt firſt know the quantity of the Angle at the Center, and that is thus diſcovered.

Seek the Complement of the verſed Sine in the Table of Natural Sines, the Degrees and Decimal parts (or minutes) thereunto belonging are the Co-ſine of half the Angle at the Center.

Thus B E 1 — G E .4 = B G .6, this found in the Table of Natural Sines gives 36 Degrees and 87 Minutes, the Angle A B F, its Complement is 53*d.* 13′. the Arch F E, which doubled is 106 *d.* 26′. the Arch of the Sector F E D, that is, the quantity of the Angle at the Centers.

Now to find the length of any number of Degrees, (that is the length of any Arch) of this Circle. Say,

As 360 the Degrees in the whole Circle, to 6.2831852 the length of the whole Circumference;

So is 1, to .01745329 the length of one Degree.

And this is a common and fixed Multiplicator for any Number of Degrees, ſo the Degrees in the Arch F E, *viz.* 53.13, being Multipled by .01745329 gives .92727787 the length of the Arch F E, that is, the Area of the Sector B F E D, which was required.

2. *To find the Area of the Triangle B F D.*

The Diameter being 2, I say half the Natural Sine of the Angle at the Center, when the said Angle does not exceed 90 *d.* when it does, then half the Natural Sine of its Complement to 180, is equal to the Area of the Triangle in the Sector.

Thus in the Sector B F E D, the Angle at the Center is 106.26 its Complement to 180. is 73.74, whose Natural Sine by the Table is .96 the half of this is .48 the Area of the Triangle B F D which was required.

The truth of which doth thus appear, B F is 1. B G is .6 *ergo* G F is .8 for B F q — B G q = F G q (by 47.1 of *Euclid.*)

But F G .8 Multiplied by B G .6 gives .48 the Area of the Triangle B F D as before, Then

From the Area of the Sector B F E D — .92727787
Take the Area of the Triangle B F D — .48000000

There remains the Area of the Segment F E D G — .44727787

But seeing in all Tables shewing the Areas of the Segments of a Circle it is supposed the Area of the whole Circle is Unity, the foregoing Table was made by this proportion.

As the Area of that Circle whose Diameter is 2, *viz.* 3.1415926, is to the Segment of any part of that Diameter:

So is 1. (the supposed Area of another Circle) to the Segment of the like part of its Diameter. Thus

As 3.1415926, is to .4472778 the Segment last found,

So is 1 to .14237, the Segment answering to the versed Sine 20 in the Table,

For

For as the verſed Sine of the firſt Segment, *viz.* G E was .4 that is $\frac{2}{5}$ of the Radius B E:

So 20 the verſed Sine of this laſt Segment is $\frac{2}{5}$ of the Radius of the Table.

The Uſe of the Table of Segments

Is to find the Ullage or Quantity of Liquor remaining in a Cask, whoſe Axis is poſited Parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Cask, and for this the Rule is;

To the wet or dry Inches of the Bung Diameter add a competent number of Cyphers, then divide it by the whole Diameter, the Quotient found in the Table under the Title V S gives a Segment, which Multiplied by the whole Content of the Cask, the Product ſhews the quantity of Liquor in the Cask, if the dividend were the wet Inches, or the Ullage if it were the dry.

Example.

Let there be a Cask in the form of a Cylinder, whoſe Bung Diameter is 33 Inches, the dry part 12, and the Content of the Cask 108 Gallons, how many Gallons are wanting to fill up the Cask?

12 divided by 33 quotes .36, which found in the Table under V S gives .2341, this Segment Multiplied by 108 (the whole Content of the Cask) the Product is 35, and ſo many Gallons the Cask wants of being full.

Note, If a Cask be in the form of a Cylinder, or near that Figure, the Table will give the Ullage more exactly than the Line of Segments on the Rule: but if the Bung Diameter be much greater than the Head Diameter, the Line of Segments is truer than the Table.

Problem VII. Figure 23.

There is a Cask supposed to be the middle Frustum of a Spheroid, standing upon one Head, with its Axis Perpendicular to the Horizon, the Dimensions

are { E H Length ——— } 24
A B Bung Diameter— } 20 } Inches.
C D Head Diameter— } 16 }

To Calculate any Diameter betwixt the Bung and Head (suppose S V) and by such Diameter to find the quantity of Liquor in the Cask when part is empty.

1. *To find the Diameter the Rule is,*

From the Square of the Bung Diameter (A B) Subtract the Square of the Head Diameter (C D) and Divide the Square-Root of the remainder by half the Casks length; (E O) then Multiply this Quotient by the Number of Inches, that the Diameter sought is distant from the Bung, and call this Product your Subducend. Lastly, from the Square of the Bung Diameter, Subtract the Square of your Subducend, the Square-Root of the remainder is the Diameter sought.

Example.

Let it be required to find the Diameter S V, which is supposed to be 6 Inches distant from A B the Bung Diameter. The

The Square of the { Bung Diameter is —— 400
Head Diameter is —— 256

The remainder is —— 144

Whose Square-Root is 12, this Divided by half the Casks Length, *viz.* 12, the Quotient is 1, this Multiplied by 6, gives 6 for the Subducend.

Square of the Bung Diameter —— 400
Square of the Subducend —— 36

There remains —— 364

Which is the Square of the Diameter S V, its Square-Root is 19.07878 the Diameter it self, but the Square of it is sufficient for our purpose.

2. *To find the Quantity of Liquor in the Cask.*

Having found the Square of S V, the Diameter of the Liquors Surface, the quantity of Liquor is easily attained:

For by the Rule laid down in the first Problem of this Section, the whole Content of the Cask will be found to be 23.532 Ale Gallons, the half of this is 11.766, for the Content of the lower half, A n t B, and by the Rule now cited, the Content of the part A S V B will be found to be 6.484 Gallons.

For the Square of A B is	400
The Square of S V is	364
Sum is	764
Half Sum is	382
Half the difference of the Square is	18
Sum of these is	1164

This Multiplied by *o g*, *viz.* 6, gives 6984, and this Divided by 1077 quotes 6.484, the Content of the part A S V B, as aboveſaid; now this added to the Content of the lower half, *viz.* 11.766, gives 18.25, and ſo many Gallons are in the Cask, if this be taken from 23.532, the whole Content, there reſts 5.282, and ſo many Gallons are required to fill up the Cask, for this 5.282 Gallons, is the Content of the part S C D V.

If the Cask were leſs than half full, the Ullage might be found by the ſame method, and that Subtracted from the whole Content leaves the quantity of Liquor in the Cask.

SECT.

Sect. X.

Of Gaging of Worts in open Veſſels.

FOR this purpoſe there are two Lines placed on the under ſide of the two Sliding Pieces of the ſhort Rule, which being ſo plain and obvious as not to need any Deſcription, I ſhall proceed to ſhew their Uſe, which is to find how many Ale Gallons, and Hundred parts of a Gallon, any ſmall Tub or ſuch like open Veſſel (from 12 to 36 Inches Diameter) will contain at one Inch deep, to perform which, obſerve the following Directions.

☞ Let the Sliding Piece C D, be pin'd faſt at the end towards the Right-Hand, then holding the Rule Parallel, draw out the other Sliding Piece 'till the two ends touch any two oppoſite ſides of the Tub or Veſſel, this done look what diviſions (both upon the Line of Gallons and Line of Inches upon the Sliding Piece) are cut by the end of the Rule, for that point doth ſhew the Diameter of the Tub in Inches and tenth parts upon one Line, and upon the other how many Gallons and Hundred parts it will contain at one Inch deep.

Thus when the end of the Rule cuts 13.4 on the Line of Inches, it will alſo cut 0 Gallons 5 tenths on the Line of Gallons; in like manner, when it cuts 21 on the Inch Line it will at the ſame time cut 1.23 on the Line of Gallons; that is, 1 Gallon and .23 Hundred parts, and ſo much will a Tub of 21 Inches Diameter contain at one Inch deep. And thus you may draw out this Piece to 24 Inches and an half,

and then the end of the Rule will cut 1.673 on the Line of Gallons.

Now if the Diameter of the Tub be more than 24.5, pin this Piece at 24.5, and then draw out the other Sliding Piece at the other end, where you will find that when the end of the Rule cuts 30 Inches, it will alſo cut 2.51 Gallons, and thus you may draw out this Piece to 36 Inches, where you will find 3 Gallons, 6 tenths and more: *Note*, In all theſe caſes 'tis the Area only which is ſought, the Inches do alſo ſhew the Diameter, but this is to make the other more plain.

Now the Tubs or Veſſels which Victualers do commonly make uſe of in cooling their Worts, and working their Ale, are either Cylindrical or Conical, and in either of theſe the Baſes are Circular or Elliptical.

Firſt of ſuch as are Cylinders or near that form.

If the Baſes of a Cylindrical Tub be Circular, the Rule doth ſhew the Area or Content at one Inch deep by inſpection, (as above) this Multiplied by the depth of the Liquor gives the Content.

For Inſtance.

Admit I come to a Tub, and (by the Rule) find the Area to be 1 Gallon 4 tenths, and the depth of the Wort 7 Inches and an half:

Multiply

Multiply the Area	1.4
by the depth, *viz.*	7.5
	70
	98
The Product is	10.50

That is, 10 Gallons and an half, the Content ſought.

But this Content may be found without the Pen, for having the Area and the depth, you may Multiply the one by the other, by the Rule: Thus,

Set 1 upon B, to 1.4 the Area upon A; then againſt 7.5 the depth upon B, you have 10.5 upon A, the Content as before.

If the Veſſel be Elliptical, take the Croſs Diameters in the middle of the depth, as ſuppoſe the longeſt Diameter is 60 Inches, and the ſhorteſt 40; Say,

As 359 is to one of the Diameters,

So is the other Diameter to the Area or Content.

And by this Proportion on the Lines A and B the Area of the Ellipſis above mentioned will be found to be 6.68: For,

As 359 upon B is to 40 upon A,

So is 60 upon B to 6.68 upon A.

This is the true way of finding the Area of an Elſipſis. But in ſmall Elliptical Tubs you may find the Area with leſs trouble:

Thus,

Find (by drawing out the Rule) the Area of the longeſt Diameter, and in like manner the Area of the ſhorteſt Diameter, half the Sum of theſe two is the Area ſought.

For Instance.

Suppose the Area of the longest Diameter be—3.62
And the Area of the shortest———→2.51
The Sum is———6.13
The half is———3.065

The Area required; which is very near the truth, for if you Work by the former Rule, you will find the true Area to be but 3.008.

Secondly for Conical Vessels.

When the Diameters at top and bottom differ not above 4 Inches, you may find the Area in the middle of the depth by drawing out the Rule as above directed, and this Area thus found is very near a true mean for the whole Tub.

Example.

Let the Diameter at the bottom of a Conical Tub be 24 Inches, and the Diameter at the Top 28 Inches, in the middle of the depth of this Tub you will find (by the Rule) the Diameter 26 Inches, and the Area 1.88 Gallons.

And for such Conical Tubs (or Tuns) whose Diameters are above 36 Inches, if they be near a Cylinder one mean Inch will be sufficient.

But for such as are much Conical and great, you may find the whole Content by Problem V. Sect. VII. And if you desire to Inch any such Tun, observe the directions in Problem I. Sect. VIII.

There

There are yet another ſort of Veſſels which Victuallers do frequently uſe to work their Ale in, and theſe are part of a But or Pipe ſet upon one Head, the other being cut off.

The whole Content of theſe may be found by Problem VII. Section IX. And how to Inch ſuch Veſſels ſhall be ſhewn in the Appendix.

I have obſerved in ſeveral places they Cool their Worts in Braſs-Pans whoſe Diameter is about 30 Inches, and depth 10 or 12 Inches, but theſe are commonly deeper in the middle by an Inch or two than at the ſide.

In ſuch Veſſels as theſe if you take (with the Rule) the Area of the Liquors Surface, and Multiply it by the depth taken within 3 or 4 Inches of the ſide, the Product will be the Content very near the truth.

And for Round Bowls which are like the Segment of a Globe, if you Multiply the Area of the Liquors Surface by half the depth, it gives the Content near enough in Practice: But in ſuch Caſes as theſe, Experience will be your beſt Tutor.

But that I may not be wanting to the meaneſt Capacities; I have here inſerted a very uſeful Table, ſhewing the Area's of Circles and Contents of Cylinders in Ale Gallons, to all Diameters in Inches and tenths, from 12 to 156 Inches Diameter, and at any depth whatſoever.

The Table explained.

The two firſt Pages (and ſo every two next ſucceeding) contains 10 Columns, in the firſt of which you have the Diameters of Circles, beginning at 12, and continued down (in the firſt Column of each left-hand Page) in Inches and tenths to 144 Inches:

The

The other 9 Columns are Numbered at the top 1, 2, 3, 4, 5, 6, 7, 8, 9. in the first of these you have the Area's of Circles at one Inch deep, to the ten thousandth part of an Ale Gallon.

In the next Column under 2, is the Content of 2 Inches deep, under 3 you have the Content at 3 Inches deep, in the next at 4 Inches deep; in like manner under 5, is the Content at 5 Inches deep, and so on to 9 Inches deep, in the last Column under 9.

The Uses of this Table are,

1. *To find the Area of any Circle, the Diameter being given.*

And this it doth by inspection, for in the Column under 1, you have the Area to any Diameter from 12 to 144: thus against this Diameter 43 in the first Column you have 5.1496 in the next Column, which is the Area sought, the like for any other.

2. *To find the Content of any Cylindrical Tun, the Diameter and depth being given.*

To effect this observe, when the depth is whole Inches only, and less than 10, you have the answer at once by inspection, for against the Diameter, and under the depth is the Content sought.

Example.

Let the Diameter of a Cylindrical Tun be 58.5 *Inches, and the depth* 6 *Inches: What is the Content?*

Seek 58 in the first Column, and in the fifth Line under it against 5 tenths, and under 6 (the depth) you

you will find 57.19, which is the Content ſought.

In like manner in the ſame Line under 7, you have 66.72, the Content at 7 Inches deep.

Note, If this Tun were 70 Inches deep the ſame Number, *viz.* 66.72 gives the Content, removing the prick one place towards the right Hand, and then it will be 667.2 Gallons.

And if the depth were but 7 tenth parts of an Inch, you muſt remove the prick one place towards the left Hand, and then it will be 6.672.

Hence 'tis evident this Table being made to 9 Inches deep, will ſhew the Content at any other depth whatſoever.

Example.

There is a Tun in the form of a Cylinder, the Diameter is 61 *Inches, and the depth* 35.6 *Inches: What is the Content?*

Find 61 in the firſt Column, and in the Line againſt it, under 3 you have 31.09, which by removing the prick is 310.9 the Content of 30 Inches deep, ſet this down as in the Margin.

Again,

In the ſame Line (againſt 61, and) under 5 is 51.81, ſet this down under the former, then for the 6 tenths take the Number under 6, (*viz.* 62.18) and removing the prick one place towards the left Hand ſet it under the other, theſe being added together:

30	310.9
5	51.81
0.6	6.218
The Sum is 35.6	= 368.928

which is the Content sought: and after the same manner you may find that a Tun of 61 Inches Diameter, and 28.5 Inches deep will contain 295.38 Gallons: See the Work:

20	207.3
8	82.9
0.5	5.18
28.5	= 295.38

A

TABLE

OF THE

AREA's of CIRCLES

And Contents of

CYLINDERS

IN

ALE GALLONS.

To all Diameters (in Inches and Tenths) from twelve Inches to one hundred fifty and six.

Diam. in Inches.	DEPTH.			
	1	2	3	4
12.0	0.4011	0.80	1.20	1.60
.1	0.4078	0.81	1.22	1.63
.2	0.4146	0.83	1.24	1.65
.3	0.4214	0.84	1.26	1.69
.4	0.4282	0.86	1.28	1.71
.5	0.4352	0.87	1.30	1.74
.6	0.4422	0.88	1.33	1.77
.7	0.4492	0.90	1.35	1.80
.8	0.4563	0.91	1.37	1.82
.9	0.4635	0.92	1.39	1.85
13.0	0.4707	0.94	1.41	1.88
.1	0.4780	0.96	1.43	1.91
.2	0.4853	0.97	1.46	1.94
.3	0.4927	0.98	1.48	1.97
.4	0.5001	1.00	1.50	2.00
.5	0.5079	1.01	1.52	2.03
.6	0.5151	1.03	1.55	2.06
.7	0.5227	1.04	1.57	2.09
.8	0.5304	1.06	1.59	2.12
.9	0.5381	1.08	1.61	2.15
14.0	0.5459	1.09	1.64	2.18
.1	0.5537	1.11	1.66	2.21
.2	0.5616	1.12	1.68	2.24
.3	0.5695	1.14	1.71	2.28
.4	0.5775	1.15	1.73	2.31
.5	0.5856	1.17	1.76	2.34
.6	0.5937	1.19	1.78	2.37
.7	0.6018	1.20	1.80	2.40
.8	0.6100	1.22	1.83	2.44
.9	0.6183	1.24	1.85	2.47

DEPTH.				
5	6	7	8	9
2.01	2.41	2.81	3.21	3.61
2.04	2.44	2.85	3.26	3.67
2.07	2.49	2.90	3.31	3.73
2.11	2.53	2.95	3.37	3.79
2.14	2.57	3.00	3.43	3.86
2.18	2.61	3.05	3.48	3.92
2.21	2.65	3.10	3.54	3.98
2.24	2.69	3.14	3.59	4.04
2.28	2.73	3.19	3.65	4.10
2.32	2.78	3.24	3.71	4.17
2.35	2.82	3.30	3.77	4.24
2.39	2.87	3.35	3.82	4.30
2.43	2.91	3.39	3.88	4.36
2.46	2.95	3.44	3.94	4.48
2.50	3.00	3.50	4.00	4.50
2.53	3.04	3.55	4.06	4.56
2.58	3.09	3.61	4.12	4.63
2.61	3.13	3.65	4.18	4.70
2.65	3.18	3.71	4.24	4.77
2.69	3.23	3.77	4.30	4.84
2.73	3.27	3.82	4.36	4.91
2.76	3.32	3.87	4.42	4.98
2.81	3.37	3.93	4.49	5.05
2.84	3.41	3.98	4.55	5.12
2.88	3.46	4.04	4.62	5.19
2.93	3.51	4.10	4.68	5.26
2.96	3.56	4.15	4.74	5.34
3.01	3.61	4.21	4.81	5.41
3.05	3.66	4.27	4.88	5.49
3.09	3.71	4.33	4.94	5.56

Diam. in Inches.	DEPTH.			
	1	2	3	4
15.0	0.6266	1.25	1.88	2.50
.1	0.6350	1.27	1.91	2.54
.2	0.6435	1.29	1.93	2.57
.3	0.6520	1.30	1.96	2.61
.4	0.6605	1.32	1.98	2.64
.5	0.6691	1.34	2.01	2.68
.6	0.6778	1.35	2.03	2.71
.7	0.6865	1.37	2.06	2.74
.8	0.6953	1.39	2.09	2.78
.9	0.7041	1.41	2.11	2.82
16.0	0.7130	1.43	2.14	2.85
.1	0.7219	1.44	2.16	2.89
.2	0.7309	1.46	2.19	2.92
.3	0.7400	1.48	2.22	2.96
.4	0.7491	1.50	2.25	3.00
.5	0.7582	1.52	2.27	3.03
.6	0.7654	1.53	2.30	3.07
.7	0.7767	1.55	2.33	3.10
.8	0.7861	1.57	2.36	3.14
.9	0.7954	1.59	2.39	3.18
17.0	0.8049	1.61	2.41	3.22
.1	0.8144	1.63	2.44	3.26
.2	0.8239	1.64	2.47	3.29
.3	6.8335	1.67	2.50	3.33
.4	0.8432	1.69	2.53	3.37
.5	0.8529	1.70	2.56	3.41
.6	0.8627	1.72	2.58	3.45
.7	0.8725	1.74	2.62	3.49
.8	0.8824	1.76	2.65	3.53
.9	0.8924	1.78	2.68	3.57

DEPTH.				
5	6	7	8	9
3.13	3.76	4.38	5.01	5.63
3.18	3.81	4.45	5.08	5.72
3.22	3.86	4.50	5.15	5.79
3.26	3.91	4.56	5.22	5.87
3.30	3.96	4.62	5.28	5.94
3.35	4.01	4.68	5.35	6.02
3.39	4.06	4.74	5.42	6.09
3.43	4.12	4.80	5.49	6.18
3.48	4.17	4.87	5.56	6.26
3.52	4.22	4.93	5.63	6.34
3.57	4.28	4.99	5.70	6.42
3.61	4.33	5.05	5.78	6.50
3.65	4.39	5.12	5.85	6.58
3.70	4.44	5.18	5.92	6.66
3.75	4.49	5.24	5.99	6.74
3.79	4.55	5.31	6.06	6.82
3.84	4.60	5.37	6.14	6.90
2.88	4.66	5.43	6.21	6.99
3.93	4.71	5.50	6.28	7.07
3.98	4.77	5.57	6.36	7.16
4.02	4.83	5.63	6.44	7.24
4.07	4.88	5.70	6.51	7.33
4.12	4.94	5.77	6.59	7.42
4.17	5.00	5.83	6.66	7.50
4.22	5.06	5.90	6.74	7.59
4.26	5.11	5.97	6.82	7.68
4.31	5.17	6.03	6.90	7.76
4.36	5.23	6.10	6.98	7.85
4.41	5.29	6.17	7.06	7.94
4.46	5.35	6.24	7.14	8.03

Diam. in Inches.	DEPTH.			
	1	2	3	4
18.0	0.9024	1.80	2.71	3.61
.1	0.9124	1.82	2.74	3.65
.2	0.9225	1.84	2.77	3.69
.3	0.9327	1.87	2.80	3.73
.4	0.9429	1.88	2.83	3.77
.5	0.9532	1.91	2.86	3.81
.6	0.9635	1.93	2.89	3.85
.7	0.9739	1.95	2.92	3.90
.8	0.9843	1.97	2.95	3.94
.9	0.9948	1.99	2.98	3.98
19.0	1.0054	2.01	3.02	4.02
.1	1.0160	2.03	3.05	4.06
.2	1.0267	2.05	3.08	4.11
.3	1.0374	2.07	3.11	4.15
.4	1.0482	2.10	3.14	4.19
.5	1.0590	2.12	3.18	4.24
.6	1.0699	2.14	3.21	4.28
.7	1.0808	2.16	3.24	4.32
.8	1.0918	2.18	3.27	4.37
.9	1.1029	2.21	3.31	4.41
20.0	1.1140	2.23	3.34	4.46
.1	1.1252	2.25	3.38	4.50
.2	1.1364	2.27	3.41	4.55
.3	1.1477	2.29	3.44	4.59
.4	1.1590	2.32	3.48	4.64
.5	1.1704	2.34	3.51	4.68
.6	1.1819	2.36	3.54	4.73
.7	1.1934	2.39	3.58	4.77
.8	1.2049	2.41	3.61	4.82
.9	1.2165	2.43	3.65	4.86

DEPTH.				
5	6	7	8	9
4.51	5.41	6.31	7.22	8.12
4.56	5.47	6.38	7.30	8.21
4.61	5.53	6.45	7.38	8.30
4.66	5.59	6.53	7.46	8.39
4.71	5.66	6.60	7.54	8.49
4.77	5.72	6.67	7.63	8.58
4.82	5.78	6.74	7.71	8.67
4.87	5.85	6.82	7.79	8.77
4.92	5.90	6.89	7.87	8.86
4.97	5.97	6.96	7.95	8.95
5.03	6.03	7.04	8.04	9.05
5.08	6.10	7.11	8.13	9.14
5.13	6.16	7.18	8.21	9.23
5.19	6.22	7.26	8.30	9.34
5.24	6.29	7.34	8.38	9.43
5.30	6.35	7.41	8.47	9.53
5.35	6.42	7.49	8.56	9.63
5.40	6.49	7.57	8.65	9.73
5.46	6.55	7.64	8.74	9.83
5.51	6.62	7.72	8.82	9.93
5.57	6.68	7.80	8.91	10.03
5.63	6.75	7.88	9.00	10.13
5.68	6.82	7.95	9.09	10.23
5.74	6.88	8.03	9.18	10.33
5.80	6.95	8.11	9.27	10.43
5.85	7.02	8.19	9.36	10.53
5.91	7.09	8.27	9.46	10.64
5.97	7.16	8.35	9.54	10.74
6.02	7.23	8.43	9.64	10.84
6.08	7.30	8.51	9.73	10.94

Diam. in Inches.	DEPTH.			
	1	2	3	4
21.0	1.2282	2.46	3.68	4.91
.1	1.2400	2.48	3.72	4.96
.2	1.2517	2.50	3.75	5.01
.3	1.2635	2.53	3.79	5.05
.4	1.2754	2.55	3.83	5.10
.5	1.2874	2.57	3.86	5.15
.6	1.2994	2.60	3.90	5.20
.7	1.3114	2.62	3.39	5.25
.8	1.3236	2.65	3.97	5.29
.9	1.3357	2.67	4.01	5.34
22.0	1.3480	2.70	4.04	5.39
.1	1.3602	2.72	4.08	5.44
.2	1.3726	2.75	4.12	5.49
.3	1.3850	2.77	4.16	5.54
.4	1.3974	2.79	4.19	5.59
.5	1.4099	2.82	4.23	5.64
.6	1.4225	2.84	4.27	5.69
.7	1.4351	2.87	4.31	5.74
.8	1.4478	2.90	4.34	5.79
.9	1.4605	2.92	4.38	5.84
23.0	1.4733	2.95	4.42	5.89
.1	1.4861	2.97	4.46	5.94
.2	1.4990	3.00	4.50	6.01
.3	1.5120	3.02	4.54	6.05
.4	1.5250	3.05	4.58	6.10
.5	1.5380	3.08	4.61	6.15
.6	1.5511	3.10	4.65	6.20
.7	1.5643	3.13	4.69	6.26
.8	1.5775	3.15	4.73	6.31
.9	1.5908	3.18	4.77	6.36

DEPTH.				
5	6	7	8	9
6.14	7.37	8.60	9.82	11.05
6.20	7.44	8.68	9.92	11.16
6.26	7.51	8.76	10.01	11.26
6.32	7.58	8.84	10.11	11.37
6.38	7.65	8.93	10.20	11.48
6.44	7.72	9.01	10.30	11.59
6.50	7.80	9.09	10.39	11.69
6.56	7.87	9.18	10.49	11.80
6.62	7.94	9.26	10.59	11.91
6.68	8.01	9.35	10.68	12.02
6.74	8.09	9.44	10.78	12.13
6.80	8.16	9.52	10.88	12.24
6.86	8.23	9.61	10.98	12.35
6.93	8.31	9.70	11.08	12.47
6.99	8.38	9.78	11.18	12.58
7.05	8.46	9.87	11.28	12.69
7.11	8.53	9.96	11.38	12.80
7.18	8.61	10.05	11.48	12.92
7.24	8.68	10.13	11.58	13.02
7.30	8.76	10.22	11.68	13.14
7.37	8.84	10.31	11.78	13.26
7.43	8.92	10.40	11.89	13.37
7.50	8.99	10.49	11.99	13.49
7.56	9.07	10.58	12.10	13.61
7.63	9.15	10.68	12.20	13.73
7.69	9.23	10.77	12.30	13.84
7.76	9.31	10.86	12.41	13.96
7.82	9.39	10.95	12.51	14.08
7.89	9.46	11.04	12.62	14.20
7.95	9.54	11.13	12.72	14.31

Diam. in Inches.	DEPTH.			
	1	2	3	4
24.0	1.6042	3.21	4.81	6.42
.1	1.6176	3.23	4.85	6.47
.2	1.6310	3.26	4.89	6.52
.3	1.6445	3.29	4.93	6.58
.4	1.6581	3.32	4.97	6.63
.5	1.6717	3.34	5.01	6.69
.6	1.6854	3.37	5.06	6.74
.7	1.6991	3.40	5.10	6.80
.8	1.7129	3.43	5.14	6.85
.9	1.7267	3.45	5.18	6.91
25.0	1.7406	3.48	5.22	6.96
.1	1.7546	3.51	5.26	7.02
.2	1.7686	3.54	5.31	7.07
.3	1.7827	3.57	5.35	7.13
.4	1.7968	3.59	5.39	7.19
.5	1.8110	3.62	5.43	7.24
.6	1.8252	3.65	5.48	7.30
.7	1.8395	3.68	5.52	7.36
.8	1.8538	3.71	5.56	7.42
.9	1.8682	3.74	5.60	7.47
26.0	1.8827	3.77	5.65	7.53
.1	1.8972	3.79	5.69	7.59
.2	1.9117	3.82	5.73	7.65
.3	1.9264	3.85	5.78	7.71
.4	1.9411	3.88	5.82	7.76
.5	1.9558	3.91	5.87	7.82
.6	1.9706	3.94	5.91	7.88
.7	1.9854	3.97	5.96	7.94
.8	2.0003	4.00	6.00	8.00
.9	2.0153	4.03	6.05	8.06

DEPTH.				
5	6	7	8	9
8.02	9.62	11.23	12.83	14.44
8.09	9.70	11.32	12.93	14.55
8.16	9.79	11.42	13.05	14.68
8.22	9.86	11.51	13.15	14.80
8.29	9.95	11.61	13.26	14.92
8.36	10.03	11.70	13.37	15.04
8.43	10.11	11.80	13.48	15.17
8.50	10.19	11.89	13.59	15.29
8.56	10.28	11.99	13.70	15.42
8.63	10.36	12.08	13.81	15.54
8.70	10.44	12.18	13.92	15.67
8.77	10.53	12.28	14.03	15.79
8.84	10.61	12.38	14.15	15.91
8.91	10.70	12.47	14.26	16.04
8.98	10.78	12.57	14.37	16.17
9.06	10.87	12.68	14.49	16.30
9.13	10.95	12.78	14.60	16.43
9.20	11.04	12.88	14.71	16.55
9.27	11.12	12.98	14.83	16.69
9.34	11.21	12.08	14.94	16.81
9.41	11.29	13.18	15.06	16.94
9.49	11.38	13.28	15.18	17.07
9.56	11.47	13.38	15.29	17.20
9.63	11.56	13.48	15.41	17.34
9.70	11.65	13.59	15.53	17.47
9.78	11.73	13.69	15.65	17.60
9.85	11.82	13.79	15.76	17.74
9.93	11.91	13.90	15.88	17.87
10.00	12.00	14.00	16.00	18.00
10.08	12.09	14.11	16.12	18.14

Diam. in Inches.	DEPTH. 1	2	3	4
27.0	2.0303	4.06	6.09	8.12
.1	2.0454	4.09	6.14	8.18
.2	2.0605	4.12	6.18	8.24
.3	2.0757	4.15	6.23	8.30
.4	2.0909	4.18	6.27	8.36
.5	2.1062	4.21	6.32	8.42
.6	2.1215	4.24	6.36	8.48
.7	2.1369	4.27	6.41	8.55
.8	2.1524	4.30	6.46	8.61
.9	2.1679	4.33	6.50	8.67
28.0	2.1835	4.37	6.55	8.73
.1	2.1991	4.40	6.60	8.80
.2	2.2148	4.43	6.64	8.86
.3	2.2305	4.46	6.69	8.92
.4	2.2463	4.49	6.74	8.98
.5	2.2621	4.52	6.79	9.05
.6	2.2780	4.56	6.83	9.11
.7	2.2940	4.59	6.88	9.18
.8	2.3100	4.62	6.93	9.24
.9	2.3261	4.65	6.98	9.30
29.0	2.3422	4.68	7.03	9.37
.1	2.3584	4.72	7.07	9.43
.2	2.3746	4.75	7.12	9.50
.3	2.3910	4.78	7.17	9.56
.4	2.4073	4.81	7.22	9.63
.5	2.4237	4.85	7. 7	9.69
.6	2.4401	4.88	7.32	9.76
.7	2.4567	4.91	7.37	9.83
.8	2.4732	4.94	7.42	9.89
.9	2.4899	4.98	7.47	9.96

DEPTH.				
5	6	7	8	9
10.15	12.18	14.21	16.24	18.27
10.23	12.27	14.32	16.36	18.41
10.30	12.36	14.42	16.48	18.54
10.38	12.45	14.53	16.61	18.68
10.45	12.55	14.64	16.73	18.82
10.53	12.64	14.74	16.85	18.95
10.61	12.73	14.85	16.97	19.09
10.68	12.82	14.96	17.10	19.23
10.76	12.91	15.06	17.22	19.37
10.84	13.01	15.18	17.34	19.51
10.92	13.10	15.28	17.46	19.65
11.00	13.19	15.39	17.59	19.79
11.08	13.29	15.50	17.71	19.93
11.15	13.38	15.61	17.84	20.07
11.23	13.48	15.72	17.97	20.22
11.31	13.57	15.83	18.10	20.36
11.39	13.67	15.95	18.22	20.50
11.47	13.76	16.06	18.35	20.65
11.55	13.86	16.17	18.48	20.79
11.63	13.96	16.28	18.61	20.93
11.71	14.05	16.39	18.74	21.08
11.79	14.15	16.51	18.86	21.22
11.87	14.25	16.62	18.99	21.37
11.96	14.35	16.74	19.13	21.52
12.04	14.44	16.85	19.26	21.66
12.12	14.54	16.96	19.39	21.81
12.20	14.64	17.08	19.52	21.96
12.28	14.74	17.19	19.65	22.10
12.37	14.84	17.31	19.78	22.26
12.45	14.94	17.43	19.92	22.41

Diam. in Inches.	DEPTH.			
	1	2	3	4
30.0	2.5066	5.01	7.52	10.03
.1	2.5233	5.05	7.57	10.09
.2	2.5401	5.08	7.62	10.16
.3	2.5570	5.11	7.67	10.23
.4	2.5739	5.15	7.72	10.29
.5	2.5908	5.18	7.77	10.36
.6	2.6078	5.21	7.82	10.43
.7	2.6249	5.25	7.87	10.50
.8	2.6420	5.28	7.93	10.57
.9	2.6592	5.32	7.98	10.64
31.0	2.6764	5.35	8.03	10.71
.1	2.6938	5.39	8.08	10.78
.2	2.7111	5.42	8.13	10.84
.3	2.7285	5.46	8.19	10.91
.4	2.7460	5.49	8.24	10.98
.5	2.7635	5.53	8.29	11.05
.6	2.7811	5.56	8.34	11.12
.7	2.7987	5.60	8.40	11.19
.8	2.8163	5.63	8.45	11.27
.9	2.8341	5.67	8.50	11.34
32.0	2.8519	5.70	8.56	11.41
.1	2.8698	5.74	8.61	11.48
.2	2.8877	5.78	8.66	11.55
.3	2.9057	5.81	8.72	11.62
.4	2.9237	5.85	8.77	11.69
.5	2.9418	5.88	8.83	11.77
.6	2.9599	5.92	8.88	11.84
.7	2.9781	5.96	8.93	11.91
.8	2.9963	5.99	8.99	11.98
.9	2.0146	6.03	8.04	12.06

DEPTH.				
5	6	7	8	9
12.53	15.04	17.54	20.05	22.56
12.62	15.14	17.66	20.18	22.71
12.70	15.24	17.78	20.32	22.86
12.79	15.34	17.90	20.46	23.01
12.87	15.44	18.02	20.59	23.17
12.95	15.55	18.14	20.73	23.32
13.04	15.65	18.25	20.86	23.47
13.12	15.75	18.37	21.00	23.62
13.21	15.85	18.49	21.14	23.78
13.30	15.95	18.61	21.27	23.93
13.38	16.06	18.73	21.41	24.09
13.47	16.16	18.86	21.55	24.25
13.56	16.27	18.98	21.69	24.40
13.64	16.37	19.10	21.83	24.56
13.73	16.48	19.22	21.97	24.71
13.82	16.58	19.34	22.10	24.87
13.91	16.69	19.47	22.25	25.03
13.99	16.79	19.59	22.38	25.18
14.08	16.90	19.71	22.53	25.35
14.17	17.00	19.84	22.67	25.51
14.26	17.11	19.96	22.82	25.67
14.35	17.22	20.09	22.96	25.83
14.44	17.32	20.21	23.10	25.98
14.53	17.43	20.34	23.24	26.15
14.62	17.54	20.46	23.39	26.31
14.71	17.65	20.59	23.53	26.48
14.80	17.76	20.72	23.68	26.64
14.89	17.87	20.85	23.82	26.80
14.98	17.98	20.97	23.97	26.97
15.07	18.09	21.10	24.11	27.13

Diam. in Inches.	DEPTH.			
	1	2	3	4
33.0	3.0330	6.07	9.10	12.13
.1	3.0514	6.10	9.15	12.21
.2	3.0698	6.14	9.21	12.28
.3	3.0884	6.18	9.26	12.35
.4	3.1069	6.21	9.32	12.43
.5	3.1256	6.25	9.38	12.50
.6	3.1443	6.29	9.43	12.58
.7	3.1630	6 33	9.49	12.65
.8	3.1818	6.36	9.55	12.73
.9	3.2007	6.40	9.60	12.80
34.0	3.2196	6.44	9.66	12.88
.1	3.2385	6.48	9.71	12.95
.2	3.2576	6.51	9.77	13.03
.3	3.2766	6.55	9.83	13.11
.4	3.2958	6.59	9.89	13.18
.5	3.3150	6.63	9.95	13.26
.6	3.3342	6.67	10.00	13.34
.7	3.3535	6.71	10.06	13.41
.8	3.3728	6.75	10.12	13.49
.9	3.3923	6.78	10.18	13.57
35.0	3.4117	6.82	10.24	13.65
.1	3.4312	6.86	10.29	13.73
.2	3.4508	6.90	10.35	13.80
.3	3.4705	6.94	10.41	13.88
.4	3.4902	6.98	10.47	13.96
.5	3.5099	7.02	10.53	14.04
.6	3.5297	7.06	10.59	14.12
.7	3.5496	7.10	10.65	14.20
.8	3.5605	7.14	10.71	14.28
.9	3.5894	7.18	10.77	14.36

DEPTH.				
5	6	7	8	9
15.17	18.20	21.23	24.26	27.30
15.26	18.31	21.36	24.41	27.46
15.35	18.42	21.49	24.56	27.63
15.44	18.53	21.62	24.71	27.79
15.53	18.64	21.75	24.85	27.96
15.63	18.75	21.88	25.00	28.13
15.72	18.86	22.01	25.15	28.30
15.82	18.98	22.14	25.30	28.47
15.91	19.09	22.27	25.45	28.63
16.00	19.20	22.40	25.60	28.80
16.10	19.32	22.54	25.75	28.97
16.19	19.43	22.67	25.90	29.14
16.29	19.54	22.80	26.06	29.32
16.38	19.66	22.93	26.21	29.49
16.48	19.77	23.07	26.36	29.66
16.58	19.89	23.21	26.52	29.84
16.67	20.00	23.34	26.67	30.01
16.77	20.12	23.47	26.82	30.18
16.86	20.23	23.61	26.98	30.35
16.96	20.35	23.75	27.14	30.53
17.06	25.47	23.88	27.29	30.70
17.16	20.59	24.01	27.45	30.88
17.25	20.70	24.16	27.61	31.06
17.35	20.82	24.29	27.76	31.24
17.45	20.94	24.43	27.92	31.41
17.55	21.06	24.57	28.08	31.59
17.65	21.18	24.71	28.24	31.76
17.75	21.30	24.85	28.39	31.94
17.85	21.42	24.09	28.55	32.13
17.95	21.53	25.12	28.71	32.30

Diam. in Inches.	DEPTH.			
	1	2	3	4
36.0	3.6095	7.22	10.83	14.44
.1	3.6295	7.26	10.89	14.52
.2	3.6497	7.30	10.95	14.60
.3	3.6699	7.34	11.01	14.68
.4	3.6901	7.38	11.07	14.76
.5	3.7104	7.42	11.13	14.84
.6	3.7308	7.46	11.19	14.92
.7	3.7512	7.50	11.25	15.00
.8	3.7717	7.54	11.32	15.09
.9	3.7922	7.58	11.38	15.17
37.0	3.8128	7.63	11.44	15.25
.1	3.8334	7.67	11.50	15.33
.2	3.8541	7.71	11.56	15.42
.3	3.8749	7.75	11.62	15.50
.4	3.8957	7.79	11.69	15.58
.5	3.9165	7.83	11.75	15.67
.6	3.9374	7.87	11.81	15.75
.7	3.9584	7.92	11.88	15.83
.8	3.9794	8.96	11.94	15.92
.9	4.0005	8.00	12.00	16.00
38.0	4.0217	8.04	12.06	16.09
.1	4.0428	8.09	12.13	16.17
.2	4.0641	8.13	12.17	16.24
.3	4.0854	8.17	12.26	16.34
.4	4.1068	8.21	12.32	16.43
.5	4.1282	8.26	12.38	16.51
.6	4.1496	8.30	12.45	16.60
.7	4.1712	8.34	12.51	16.68
.8	4.1927	8.39	12.58	16.77
.9	4.2144	8.43	12.64	16.86

DEPTH.				
5	6	7	8	9
18.05	21.66	25.27	28.87	32.48
18.15	21.78	25.41	29.03	32.66
18.25	21.90	25.55	29.19	32.84
18.35	22.02	25.69	29.36	33.03
18.45	22.14	25.83	29.52	33.21
18.55	22.26	25.97	29.68	33.39
18.65	22.39	26.11	29.85	33.58
18.76	22.51	26.26	30.01	33.76
18.86	22.63	26.40	30.17	33.94
18.96	22.75	26.55	30.34	34.13
19.07	22.88	26.69	30.51	34.32
19.17	23.00	26.83	30.67	34.50
19.27	23.12	26.98	30.83	34.69
19.37	23.25	27.12	31.00	34.87
19.48	23.37	27.27	31.17	35.06
19.58	23.50	27.41	31.33	35.25
19.69	23.62	27.56	31.50	35.43
19.79	23.75	27.71	31.66	35.62
19.90	23.87	27.86	31.83	35.81
20.00	24.00	28.00	32.00	36.00
20.11	24.13	28.15	32.17	36.19
20.21	24.26	28.30	32.34	36.39
20.30	24.36	28.43	32.49	36.56
20.43	24.51	28.60	32.68	36.77
20.54	24.64	28.75	32.85	36.96
20.64	24.77	28.90	33.02	37.15
20.75	24.90	29.05	33.19	37.34
20.86	25.03	29.20	33.37	37.54
20.96	25.15	29.35	33.54	37.73
21.07	25.29	29.50	33.71	37.93

Diam. in Inches.	DEPTH.			
	1	2	3	4
39.0	4.2361	8.47	12.71	16.94
.1	4.2578	8.52	12.77	17.03
.2	4.2796	8.56	12.84	17.12
.3	4.3015	8.60	12.90	17.20
.4	4.3234	8.65	12.97	17.29
.5	4.3454	8.69	13.04	17.38
.6	4.3674	8.73	13.10	17.47
.7	4.3895	8.78	13.17	17.56
.8	4.4117	8.82	13.24	17.65
.9	4.4338	8.87	13.30	17.74
40.0	4.4562	8.91	13.37	17.82
.1	4.4785	8.96	13.43	17.91
.2	4.5008	9.00	13.50	18.00
.3	4.5233	9.05	13.57	18.09
.4	4.5457	9.09	13.64	18.18
.5	4.5683	9.14	13.71	18.27
.6	4.5908	9.18	13.77	18.36
.7	4.6135	9.23	13.84	18.45
.8	4.6362	9.27	13.91	18.54
.9	4.6589	9.32	13.98	18.64
41.0	4.6818	9.36	14.05	18.73
.1	4.7046	9.41	14.11	18.82
.2	4.7275	9.45	14.18	18.91
.3	4.7505	9.50	14.25	19.00
.4	4.7735	9.55	14.32	19.09
.5	4.7966	9.59	14.39	19.19
.6	4.8198	9.64	14.46	19.28
.7	4.8430	9.69	14.53	19.37
.8	4.8662	9.73	14.60	19.46
.9	4.8895	9.78	14.67	19.56

DEPTH.				
5	6	7	8	9
21.18	25.42	29.65	33.89	38.12
21.29	25.55	29.81	34.06	38.32
21.40	25.68	29.95	34.23	38.51
21.51	25.81	30.11	34.41	38.71
21.62	25.94	30.26	34.58	38.91
21.73	26.07	30.42	34.76	39.11
21.84	26.20	30.57	34.94	39.30
21.95	26.33	30.72	35.11	39.50
22.06	26.47	30.89	35.30	39.71
22.17	26.60	31.04	35.47	39.91
22.28	26.74	31.19	35.65	40.11
22.39	26.87	31.35	35.83	40.30
22.50	27.00	31.51	36.01	40.51
22.62	27.14	31.66	36.18	40.71
22.73	27.28	31.82	36.37	40.91
22.84	27.41	31.98	36.54	41.11
22.95	27.55	32.14	36.73	41.32
23.07	27.68	32.29	36.91	41.52
23.18	27.82	32.45	37.09	41.72
23.30	27.95	32.61	37.27	41.93
23.41	28.09	32.77	37.46	42.14
23.52	28.22	32.93	37.63	42.34
23.64	28.36	33.09	37.82	42.54
23.75	28.50	33.25	38.00	42.75
23.87	28.64	33.41	38.19	42.96
23.98	28.78	33.57	38.37	43.17
24.10	28.92	33.74	38.56	43.38
24.22	29.06	33.90	38.74	43.59
24.33	29.19	34.06	38.93	43.80
24.45	29.34	34.23	39.11	44.00

Diam. in Inches.	DEPTH.			
	1	2	3	4
42.0	4.9129	9.83	14.74	19.65
.1	4.9363	9.87	14.81	19.74
.2	4.9598	9.92	14.88	19.84
.3	4.9834	9.97	14.95	19.93
.4	5.0069	10.01	15.02	20.03
.5	5.0306	10.06	15.09	20.12
.6	5.0543	10.11	15.16	20.22
.7	5.0780	10.16	15.23	20.31
.8	5.1019	10.20	15.31	20.41
.9	5.1257	10.25	15.38	20.50
43.0	5.1496	10.30	15.45	20.60
.1	5.1736	10.35	15.52	20.69
.2	5.1977	10.40	15.59	20.79
.3	5.2218	10.44	15.67	20.89
.4	5.2459	10.49	15.74	20.99
.5	5.2701	10.54	15.81	21.08
.6	5.2944	10.59	15.88	21.18
.7	5.3187	10.64	15.96	21.28
.8	5.3430	10.69	16.03	21.37
.9	5.3675	10.74	16.10	21.47
44.0	5.3920	10.78	16.18	21.57
.1	5.4165	10.83	16.25	21.67
.2	5.4411	10.88	16.32	21.76
.3	5.4657	10.93	16.40	21.86
.4	5.4904	10.98	16.47	21.96
.5	5.5152	11.03	16.55	22.06
.6	5.5400	11.08	16.62	22.16
.7	5.5649	11.13	16.70	22.26
.8	5.5898	11.18	16.77	22.36
.9	5.6148	11.23	16.85	22.46

DEPTH.				
5	6	7	8	9
24.57	29.48	34.39	39.30	44.22
24.68	29.62	34.55	39.49	44.43
24.80	29.76	34.72	39.68	44.64
24.92	29.90	34.88	39.86	44.85
25.04	30.04	35.05	40.06	45.06
25.16	30.19	35.22	40.25	45.28
25.27	30.33	35.38	40.43	45.49
25.39	30.47	35.55	40.62	45.70
25.51	30.61	35.72	40.82	45.92
25.63	30.76	35.88	41.01	46.14
25.75	30.90	36.05	41.19	46.34
25.87	31.04	36.21	41.39	46.56
25.99	31.19	36.39	41.59	46.78
26.11	31.33	36.56	41.78	47.00
26.23	31.48	36.72	41.97	47.22
26.35	31.62	36.89	42.16	47.43
26.47	31.77	37.06	42.35	47.65
26.59	31.91	37.23	42.55	47.87
26.72	32.06	37.40	42.75	48.09
26.84	32.20	37.57	42.94	48.31
26.96	32.35	37.74	43.14	48.53
27.08	32.50	37.91	43.33	48.75
27.21	32.65	37.09	43.53	48.97
27.33	32.80	38.26	43.73	49.19
27.45	32.94	38.43	43.92	49.41
27.58	33.09	38.61	44.12	49.64
27.70	33.24	38.78	44.32	49.86
27.83	33.39	38.96	44.52	50.09
27.95	33.54	39.13	44.72	50.31
28.08	33.69	39.31	44.92	50.54

Diam. in Inches.	DEPTH.			
	1	2	3	4
45.0	5.6398	11.28	16.92	22.56
.1	5.6649	11.33	17.00	22.66
.2	5.6901	11.38	17.07	22.76
.3	5.7153	11.43	17.15	22.86
.4	5.7405	11.48	17.22	22.96
.5	5.7659	11.53	17.30	23.06
.6	5.7912	11.58	17.37	23.16
.7	5.8167	11.63	17.45	23.27
.8	5.8421	11.68	17.53	23.37
.9	5.8677	11.74	17.60	23.47
46.0	5.8933	11.79	17.68	23.57
.1	5.9189	11.84	17.76	23.68
.2	5.9446	11.89	17.84	23.78
.3	5.9704	11.94	17.91	23.88
.4	5.9962	11.99	17.99	23.98
.5	6.0221	12.04	18.07	24.09
.6	6.0480	12.10	18.14	24.19
.7	6.0740	12.15	18.22	24.30
.8	6.1000	12.20	18.30	24.40
.9	6.1261	12.25	18.38	24.50
47.0	6.1523	12.30	18.46	24.61
.1	6.1735	12.36	18.54	24.71
.2	6.2048	12.41	18.61	24.82
.3	6.2311	12.46	18.69	24.92
.4	6.2575	12.52	18.78	25.03
.5	6.2839	12.57	18.85	25.14
.6	6.3104	12.62	18.93	25.24
.7	6.3369	12.67	19.01	25.35
.8	6.3635	12.73	19.09	25.45
.9	6.3902	12.78	19.17	25.56

DEPTH.				
5	6	7	8	9
28.20	33.84	39.48	45.12	50.76
28.33	33.99	39.66	45.32	50.99
28.45	34.14	39.83	45.52	51.21
28.58	34.29	40.01	45.72	51.44
28.70	34.44	40.18	45.92	51.66
28.83	34.60	40.36	46.13	51.90
28.96	34.75	40.54	46.33	52.12
29.09	34.90	40.72	46.54	52.35
29.21	35.05	40.90	46.74	52.58
29.34	35.21	41.08	46.94	52.81
29.47	35.36	41.25	47.15	53.04
29.60	35.51	41.43	47.35	53.27
29.73	35.67	41.62	47.56	53.51
29.85	35.82	41.79	47.76	53.73
29.98	35.98	41.97	47.97	53.97
30.11	36.13	42.15	48.18	54.20
30.24	36.29	42.34	48.38	54.43
30.37	36.44	42.52	48.59	54.67
30.50	36.60	42.70	48.80	54.90
30.63	36.76	42.88	49.01	55.13
30.76	36.91	43.07	49.22	55.37
30.89	37.07	43.25	49.43	55.60
31.03	37.23	43.44	49.64	55.85
31.16	37.39	43.62	49.85	56.08
31.29	37.55	43.81	50.07	56.32
31.42	37.71	43.99	50.27	56.56
31.55	37.86	44.17	50.48	56.79
31.69	38.02	44.36	50.70	57.03
31.82	38.18	44.54	50.91	57.27
31.95	38.34	44.73	51.12	57.51

Diam. in Inches.	DEPTH.			
	1	2	3	4
48.0	6.4169	12.83	19.25	25.67
.1	6.4436	12.89	19.33	25.78
.2	6.4705	12.94	19.41	25.88
.3	6.4973	12.99	19.49	25.99
.4	6.5243	13.05	19.57	26.10
.5	6.5513	13.10	19.65	26.20
.6	6.5783	13.16	19.74	26.31
.7	6.6054	13.21	19.82	26.42
.8	6.6325	13.27	19.90	26.53
.9	6.6600	13.32	19.98	26.64
49.0	6.6870	13.37	20.06	26.74
.1	6.7143	13.43	20.14	26.86
.2	6.7417	13.48	20.23	26.97
.3	6.7692	13.54	20.31	27.08
.4	6.7966	13.59	20.39	27.19
.5	6.8242	13.65	20.47	27.30
.6	6.8518	13.70	20.56	27.41
.7	6.8794	13.76	20.64	27.52
.8	6.9072	13.81	20.72	27.63
.9	6.9349	13.87	20.81	27.74
50.0	6.9628	13.93	20.89	27.85
.1	6.9906	13.98	20.97	27.96
.2	7.0186	14.04	21.06	28.08
.3	7.0466	14.09	21.14	28.19
.4	7.0746	14.15	21.22	28.30
.5	7.1027	14.21	21.31	28.41
.6	7.1309	14.26	21.39	28.52
.7	7.1591	14.32	21.48	28.64
.8	7.1873	14.38	21.56	28.75
.9	7.2157	14.43	21.65	28.86

DEPTH.				
5	6	7	8	9
32.08	38.50	44.92	51.34	57.75
32.22	38.66	45.11	51.55	58.00
32.35	38.82	45.29	51.76	58.23
32.49	38.98	45.48	51.98	58.48
32.62	39.15	45.67	52.19	58.72
32.76	39.31	45.86	52.41	58.96
32.89	39.47	46.05	52.63	59.20
33.03	39.63	46.24	52.84	59.45
33.16	39.79	46.43	53.06	59.69
33.30	39.96	46.62	53.28	59.94
33.43	40.12	46.80	53.49	60.18
33.57	40.29	47.00	53.71	60.43
33.71	40.45	47.20	53.94	60.68
33.85	40.61	47.38	54.15	60.92
33.99	40.78	47.58	54.38	60.17
34.12	40.95	47.77	54.59	61.42
34.26	41.11	47.97	54.82	61.67
34.40	41.28	48.15	55.03	61.91
34.54	41.44	48.35	55.26	62.16
34.68	41.61	48.55	55.48	62.42
34.82	41.78	48.74	55.71	62.66
34.96	41.95	48.94	55.93	62.92
35.10	42.11	49.13	56.15	63.17
35.24	42.28	49.33	56.38	63.42
35.38	42.45	49.53	56.60	63.68
35.52	42.62	49.72	56.83	63.93
35.66	42.79	49.92	57.05	64.18
35.80	42.96	50.11	57.27	64.43
35.94	43.12	50.31	57.50	64.69
36.08	43.30	50.51	57.73	64.94

Diam. in Inches.	DEPTH.			
	1	2	3	4
51.0	7.2440	14.49	21.73	28.98
.1	7.2725	14.55	21.82	29.09
.2	7.3010	14.60	21.90	29.20
.3	7.3295	14.66	21.99	29.32
.4	7.3581	14.72	22.07	29.43
.5	7.3868	14.77	22.16	29.55
.6	7.4155	14.83	22.25	29.66
.7	7.4443	14.89	22.33	29.78
.8	7.4731	14.95	22.42	29.89
.9	7.5020	15.00	22.51	30.01
52.0	7.5309	15.06	22.59	30.12
.1	7.5599	15.12	22.68	30.24
.2	7.5890	15.18	22.77	30.36
.3	7.6181	15.24	22.86	30.47
.4	7.6472	15.29	22.94	30.59
.5	7.6764	15.35	23.03	30.71
.6	7.7057	15.41	23.12	30.83
.7	7.7350	15.47	23.21	30.94
.8	7.7644	15.53	23.29	31.06
.9	7.7939	15.59	23.38	31.18
53.0	7.8233	15.65	23.47	31.29
.1	7.8529	15.71	23.56	31.41
.2	7.8825	15.76	23.65	31.53
.3	7.9122	15.82	23.74	31.65
.4	7.9419	15.88	23.83	31.77
.5	7.9717	15.94	23.92	31.89
.6	8.0015	16.00	24.00	32.01
.7	8.0314	16.06	24.09	32.13
.8	8.0613	16.12	24.18	32.24
.9	8.0913	16.18	24.27	32.36

DEPTH.				
5	6	7	8	9
36.22	43.47	50.71	57.95	65.20
36.36	43.63	50.91	58.18	65.45
36.51	43.81	51.11	58.41	65.71
36.65	43.97	51.30	58.63	65.96
36.79	44.15	51.51	58.87	66.22
36.94	44.32	51.71	59.10	66.48
37.08	44.49	51.91	59.32	66.74
37.22	44.66	52.11	59.55	67.00
37.37	44.84	52.31	59.79	67.26
37.51	45.01	52.52	60.02	67.52
37.66	45.19	52.72	60.25	67.78
37.80	45.36	52.92	60.48	68.04
37.95	45.53	53.12	60.71	68.30
38.09	45.71	53.33	60.95	68.56
38.24	45.88	53.53	61.18	68.82
38.38	46.06	53.73	61.41	69.09
38 53	46.24	53.94	61.65	69.36
38.68	46.41	54.15	61.88	69.62
38.82	46.59	54.35	62.11	69.88
38.97	46.76	54.56	62.35	70.15
39.12	46.94	54.76	62.59	70.41
39.27	47.12	54.97	62.82	70.68
39.41	47.29	55.18	63.06	70.94
39.56	47.47	55.39	63.30	71.21
39.71	47.65	55.60	63.54	71.48
39.86	47.83	55.81	63.78	71.75
40.01	48.01	56.01	64.01	72.01
40.16	48.19	56.22	64.25	72.28
40.31	48.37	56.43	64.49	72.55
40.46	48.55	56.64	64.73	72.82

Diam. in Inches.	DEPTH.			
	1	2	3	4
54.0	8.1214	16.24	24.36	32.49
.1	8.1515	16.30	24.45	32.61
.2	8.1816	16.36	24.54	32.72
.3	8.2118	16.42	24.64	32.85
.4	8.2421	16.48	24.73	32.97
.5	8.2724	16.54	24.82	33.09
.6	8.3028	16.61	24.91	33.22
.7	8.3333	16.67	25.00	33.33
.8	8.3638	16.72	25.09	33.46
.9	8.3943	16.79	25.18	33.58
55.0	8.4249	16.85	25.27	33.70
.1	8.4556	16.91	25.37	33.82
.2	8.4863	16.97	25.46	33.95
.3	8.5171	17.03	25.55	34.07
.4	8.5479	17.10	25.64	34.19
.5	8.5788	17.16	25.74	34.32
.6	8.6097	17.22	25.83	34.44
.7	8.6407	17.28	25.92	34.56
.8	8.6718	17.34	26.01	34.68
.9	8.7029	17.40	26.11	34.81
56.0	8.7341	17.47	26.20	34.94
.1	8.7653	17.53	26.30	35.06
.2	8.7966	17.59	26.39	35.19
.3	8.8279	17.66	26.48	35.31
.4	8.8593	17.72	26.58	35.44
.5	8.8907	17.78	26.67	35.56
.6	8.9222	17.84	26.77	35.69
.7	8.9538	17.91	26.86	35.82
.8	8.9854	17.97	26.96	35.91
.9	9.0171	17.03	27.05	36.07

DEPTH.				
5	6	7	8	9
40.61	48.73	56.85	64.97	73.09
40.76	48.91	57.06	65.21	73.36
40.91	49.09	57.27	65.45	73.63
41.06	49.27	57.49	65.70	73.91
41.21	49.45	57.70	65.94	74.18
41.36	49.63	55.91	66.18	74.45
41.52	49.82	58.12	66.43	74.73
41.67	50.00	58.33	66.67	75.00
41.82	50.19	58.55	66.91	75.28
41.97	50.37	58.76	67.15	75.55
42.12	50.55	58.98	67.40	75.83
42.28	50.74	59.19	67.65	76.11
42.43	50.92	59.40	67.89	76.38
42.59	51.10	59.62	68.14	76.66
42.74	51.29	59.84	68.39	76.93
42.90	51.48	60.05	68.63	77.21
43.05	51.66	60.27	68.88	77.49
43.20	51.85	60.49	69.13	77.77
43.36	52.03	60.70	69.37	77.04
43.52	52.22	60.92	69.62	77.33
43.67	52.40	61.14	69.87	78.61
43.83	52.59	61.36	70.12	78.89
43.99	52.78	61.58	70.38	79.18
44.14	52.97	61.80	70.62	79.45
44.30	52.16	62.01	70.87	79.73
44.46	53.35	62.24	71.13	80.02
44.61	53.53	62.46	71.38	80.30
44.77	53.72	62.68	71.63	80.59
44.93	53.91	62.90	71.88	80.87
45.09	54.10	63.12	72.14	81.15

Diam. in Inches.	DEPTH.			
	1	2	3	4
57.0	9.0489	18.10	27.15	36.20
.1	9.0806	18.16	27.24	36.32
.2	9.1124	18.22	27.34	36.45
.3	9.1443	18.29	27.43	36.58
.4	9.1762	18.35	27.53	36.71
.5	9.2082	18.42	27.62	36.83
.6	9.2403	18.48	27.72	36.96
.7	9.2724	18.54	27.82	37.09
.8	9.3046	18.61	27.91	37.22
.9	9.3368	18.67	28.01	37.35
58.0	9.3691	18.74	28.11	37.48
.1	9.4014	18.80	28.20	37.60
.2	9.4338	18.87	28.30	37.74
.3	9.4662	18.93	28.40	37.86
.4	9.4988	19.00	28.50	38.00
.5	9.5313	19.06	28.59	38.12
.6	9.5639	19.13	28.69	38.26
.7	9.5966	19.19	28.79	38.38
.8	9.6293	19.26	28.89	38.52
.9	9.6621	19.32	28.99	38.65
59.0	9.6949	19.39	29.08	38.78
.1	9.7278	19.46	29.19	38.91
.2	9.7608	19.52	29.28	39.04
.3	9.7938	19.59	29.38	39.18
.4	9.7268	19.65	29.48	39.31
.5	9.8600	19.72	29.58	39.44
.6	9.8931	19.79	29.68	39.57
.7	9.9263	19.85	29.78	39.70
.8	9.9596	19.92	29.88	39.84
.9	9.9930	19.99	29.98	39.97

DEPTH.				
5	6	7	8	9
45.25	54.29	63.34	72.39	81.44
45.40	54.48	63.56	72.65	81.73
45.56	54.67	63.79	72.90	82.01
45.72	54.87	64.01	73.15	82.30
45.88	55.06	64.23	73.41	82.59
46.04	55.25	64.46	73.67	82.87
46.20	55.44	64.68	73.92	83.16
46.36	55.63	64.91	74.18	83.45
46.52	55.83	65.13	74.43	83.74
46.69	56.02	65.36	74.70	84.03
46.85	56.21	65.58	74.95	84.32
47.01	56.41	65.81	75.21	84.61
47.17	56.60	66.04	75.47	84.91
47.33	56.80	66.26	75.73	85.20
47.50	56.99	66.49	75.99	85.49
47.66	57.19	66.72	76.25	85.78
47.82	57.38	66.95	76.51	86.08
47.98	57.58	67.17	76.77	86.37
48.15	57.77	67.40	77.03	86.66
48.31	57.97	67.63	77.30	86.96
48.48	58.17	67.87	77.56	87.26
48.64	58.37	68.10	77.82	87.55
48.81	58.57	68.33	78.09	87.85
48.97	58.77	68.56	78.35	88.15
49.14	58.96	68.79	78.62	88.44
49.30	59.16	69.02	78.88	88.74
49.47	59.36	69.25	79.15	89.04
49.63	59.56	69.48	79.41	89.34
49.80	59.75	69.71	79.67	89.63
49.97	59.96	69.95	79.95	89.94

Diam. in Inches.	DEPTH.			
	1	2	3	4
60.0	10.0264	20.05	30.08	40.10
.1	10.0598	20.12	30.18	40.24
.2	10.0933	20.19	30.28	40.37
.3	10.1269	20.25	30.38	40.51
.4	10.1605	20.32	30.48	40.64
.5	10.1942	20.39	30.58	40.78
.6	10.2279	20.46	30.68	40.91
.7	10.2617	20.52	30.78	41.04
.8	10.2955	20.59	30.89	41.18
.9	10.3294	20.66	30.99	41.32
61.0	10.3634	20.73	31.09	41.45
.1	10.3974	20.80	31.19	41.59
.2	10.4314	20.86	31.29	41.72
.3	10.4655	20.93	31.40	41.86
.4	10.4997	21.00	31.50	42.00
.5	10.5339	21.07	31.60	42.14
.6	10.5682	21.14	31.70	42.27
.7	10.6026	21.21	31.81	42.41
.8	10.6370	21.27	31.91	42.55
.9	10.6714	21.34	32.01	42.68
62.0	10.7059	21.41	32.11	42.82
.1	10.7405	21.48	32.22	42.96
.2	10.7751	21.55	32.33	43.10
.3	10.8098	21.62	32.43	43.24
.4	10.8445	21.69	32.53	43.38
.5	10.8793	21.76	32.64	43.52
.6	10.9141	21.83	32.74	43.66
.7	10.9490	21.90	32.85	43.80
.8	10.9840	21.97	32.95	43.94
.9	11.0190	22.04	33.06	44.08

DEPTH.				
5	6	7	8	9
50.13	60.16	70.18	80.21	90.23
50.30	60.36	70.42	80.48	90.54
50.47	60.56	70.65	80.75	90.84
50.64	60.76	70.89	81.02	91.14
50.80	60.96	71.12	81.28	91.45
50.97	61.17	71.36	81.55	91.75
51.14	61.37	71.60	81.82	92.05
51.31	61.57	71.83	82.09	92.35
51.48	61.77	72.07	82.36	92.66
51.65	61.97	72.30	82.63	92.96
51.81	62.18	72.54	82.90	92.27
51.99	62.38	72.78	83.18	93.58
52.16	62.59	73.02	83.45	93.88
52.33	62.79	73.26	83.72	93.19
52.50	63.00	73.50	84.00	94.50
52.67	63.20	73.74	84.27	94.81
52.84	63.41	73.98	84.55	95.11
53.02	63.62	74.22	84.82	95.43
53.19	63.82	74.46	85.10	95.73
53.36	64.03	74.70	85.37	96.04
53.53	64.24	74.94	85.65	96.35
53.70	64.44	75.18	85.92	96.66
53.88	64.65	75.43	86.20	96.98
54.05	64.86	75.67	86.48	97.29
54.22	65.06	75.91	86.75	97.60
54.40	65.27	76.15	87.03	97.91
54.57	65.48	76.40	87.31	98.23
54.75	65.69	76.64	87.59	98.54
54.92	65.91	76.89	87.87	98.86
55.10	66.11	77.13	88.15	99.17

Diam. in Inches.	DEPTH. 1	2	3	4
63.0	11.0541	22.11	33.16	44.22
.1	11.0892	22.18	33.27	44.36
.2	11.1244	22.25	33.37	44.50
.3	11.1596	22.32	33.48	44.64
.4	11.1949	22.39	33.59	44.78
.5	11.2302	22.46	33.69	44.92
.6	11.2656	22.53	33.79	45.06
.7	11.3011	22.60	33.90	45.20
.8	11.3366	22.67	34.01	45.35
.9	11.3721	22.74	34.12	45.49
64.0	11.4078	22.82	34.22	45.63
.1	11.4434	22.89	34.33	45.77
.2	11.4792	22.96	34.44	45.92
.3	11.5150	23.03	34.54	46.06
.4	11.5508	23.10	34.65	46.20
.5	11.5867	23.17	34.76	46.35
.6	11.6227	23.25	34.87	46.49
.7	11.6587	23.32	34.98	46.64
.8	11.6947	23.39	35.08	46.78
.9	11.7309	23.46	35.19	46.92
65.0	11.7670	23.53	35.30	47.07
.1	11.8033	23.60	35.40	47.21
.2	11.8396	23.68	35.52	47.36
.3	11.8759	23.75	35.62	47.50
.4	11.9123	23.82	35.74	47.65
.5	11.9488	23.90	35.85	47.80
.6	11.9853	23.97	35.96	47.94
.7	12.0219	24.04	36.07	48.09
.8	12.0585	24.12	36.17	48.23
.9	12.0952	24.19	36.28	48.38

DEPTH.				
5	6	7	8	9
55.27	66.32	77.38	88.43	99.49
55.45	66.53	77.62	88.71	99.80
55.62	66.75	77.87	88.99	100.12
55.80	66.96	78.12	89.28	100.44
55.98	67.17	78.37	89.56	100.76
56.15	67.38	78.61	89.84	101.07
56.33	67.60	78.86	90.13	101.40
56.51	67.81	79.11	90.41	101.71
56.69	68.02	79.36	90.70	102.03
56.86	68.23	79.60	90.98	102.35
57.04	68.45	79.86	91.26	102.67
57.22	68.66	80.10	91.54	102.99
57.40	68.87	80.35	91.83	103.31
57.57	69.09	80.61	92.12	103.64
57.75	69.31	80.86	92.41	103.96
57.94	69.52	81.11	92.70	104.28
58.12	69.74	81.36	92.98	104.61
58.30	69.95	81.61	93.27	104.93
58.47	70.17	81.87	93.56	105.26
58.65	70.39	82.12	93.85	105.58
58.84	70.60	82.37	94.14	105.90
59.01	70.81	82.62	94.42	106.22
59.20	71.04	82.88	94.72	106.56
59.38	71.26	83.13	95.01	106.88
59.56	71.47	83.38	95.30	107.21
59.75	71.69	83.64	95.60	107.54
59.93	71.91	83.90	95.88	107.87
60.11	72.13	84.15	96.18	108.20
60.29	72.35	84.41	96.46	108.52
60.48	72.57	84.67	96.76	108.86

Diam. in Inches.	DEPTH			
	1	2	3	4
66.0	12.1319	24.26	36.40	48.53
.1	12.1687	24.34	36.51	48.68
.2	12.2055	24.41	36.62	48.82
.3	12.2424	24.48	36.73	48.97
.4	12.2794	24.56	36.84	49.12
.5	12.3164	24.63	36.95	49.27
.6	12.3535	24.71	37.06	49.41
.7	12.3906	24.78	37.17	49.56
.8	12.4278	24.86	37.28	49.71
.9	12.4650	24.93	37.40	49.86
67.0	12.5023	25.00	37.51	50.01
.1	12.5397	25.08	37.62	50.16
.2	12.5771	25.15	37.73	50.31
.3	12.6145	25.23	37.84	50.46
.4	12.6520	25.30	37.96	50.61
.5	12.6896	25.38	38.07	50.76
.6	12.7272	25.45	38.18	50.91
.7	12.7649	25.53	38.30	51.06
.8	12.8027	25.61	38.41	51.21
.9	12.8405	25.68	38.52	51.36
68.0	12.8783	25.76	38.63	51.51
.1	12.9162	25.83	38.75	51.66
.2	12.9542	25.91	38.86	51.82
.3	12.9922	25.98	38.98	51.97
.4	13.0303	26.06	39.09	52.12
.5	13.0684	26.14	39.20	52.27
.6	13.1066	26.21	39.32	52.42
.7	13.1448	26.29	39.44	52.58
.8	13.1831	26.37	39.55	52.73
.9	13.2215	26.44	39.66	52.88

DEPTH.				
5	6	7	8	9
60.66	72.79	84.92	97.06	109.19
60.85	73.01	85.18	97.35	109.52
61.03	73.23	85.44	97.64	109.85
61.21	73.45	85.69	97.94	110.18
61.40	73.67	85.95	98.23	110.51
61.58	73.90	86.21	98.53	110.84
61.77	74.12	86.47	98.83	111.18
61.95	74.34	86.73	99.12	111.51
62.14	74.57	87.00	99.42	111.85
62.33	74.79	87.26	99.72	112.19
62.51	75.01	87.51	100.02	112.52
62.70	75.24	87.78	100.32	112.86
62.89	75.46	88.04	100.62	113.19
63.07	75.68	88.30	100.91	113.53
63.26	75.91	88.56	101.22	113.87
63.45	76.13	88.82	101.51	114.20
63.64	76.36	89.09	101.82	114.54
63.83	76.59	89.36	102.12	114.89
64.02	76.82	89.62	102.42	115.23
64.20	77.04	89.88	102.72	115.56
64.39	77.27	90.14	103.02	115.90
64.58	77.50	90.41	103.33	116.24
64.77	77.72	90.68	103.63	116.59
64.96	77.95	90.94	103.94	116.93
65.15	78.18	91.21	104.24	117.27
65.34	78.41	91.48	104.54	117.61
65.53	78.64	91.74	104.85	117.95
65.73	78.87	92.02	105.16	118.31
65.92	79.10	92.28	105.47	118.65
66.11	79.33	92.55	105.77	118.90

Diam. in Inches.	DEPTH.			
	1	2	3	4
69.0	13.2599	26.52	39.78	53.04
.1	13.2983	26.60	39.89	53.19
.2	13.3368	26.67	40.01	53.35
.3	13.3754	26.75	40.13	53.50
.4	13.4140	26.83	40.24	53.66
.5	13.4527	26.91	40.36	53.81
.6	13.4915	26.98	40.47	53.96
.7	13.5303	27.06	40.59	54.12
.8	13.5691	27.14	40.71	54.28
.9	13.6080	27.22	40.82	54.43
70.0	13.6470	27.29	40.94	54.59
.1	13.6860	27.37	41.06	54.74
.2	13.7251	27.45	41.18	54.90
.3	13.7642	27.53	41.29	55.06
.4	13.8034	27.61	41.41	55.21
.5	13.8426	27.68	41.53	55.37
.6	13.8819	27.76	41.65	55.53
.7	13.9213	27.84	41.76	55.68
.8	13.9607	27.92	41.88	55.84
.9	14.0002	28.00	42.00	56.00
71.0	14.0397	28.08	42.12	56.16
.1	14.0793	28.16	42.24	56.32
.2	14.1189	28.24	42.36	56.48
.3	14.1586	28.32	42.47	56.63
.4	14.1983	28.40	42.59	56.79
.5	14.2381	28.48	42.71	56.95
.6	14.2780	28.56	42.83	57.11
.7	14.3179	28.64	42.95	57.27
.8	14.3579	28.72	43.07	57.43
.9	14.3979	28.80	43.19	57.59

DEPTH.				
5	6	7	8	9
66.30	79.56	92.82	106.08	119.34
66.49	79.79	93.09	106.38	119.68
66.69	80.02	93.36	106.70	120.03
66.88	80.25	93.63	107.00	120.38
67.07	80.48	93.90	107.31	120.73
67.27	80.72	94.17	107.62	121.08
67.46	80.95	94.44	107.93	121.42
67.65	81.18	94.71	108.24	121.77
67.85	81.41	94.98	108.55	122.12
68.04	81.65	95.26	108.86	122.47
68.24	81.88	95.53	109.18	122.82
68.43	82.12	95.80	109.49	123.17
68.63	82.35	96.08	109.80	123.53
68.82	82.58	96.35	110.11	123.88
69.02	82.82	96.62	110.42	124.23
69.21	83.05	96.89	111.74	124.58
69.41	83.29	97.17	111.06	124.94
69.61	83.53	97.45	111.37	125.29
69.81	83.77	97.73	111.69	125.65
70.00	84.00	98.00	112.00	126.00
70.20	84.24	98.28	112.32	126.36
70.40	84.47	98.55	112.63	126.71
70.60	84.71	98.83	112.95	127.07
70.79	84.95	99.11	113.26	127.42
70.99	85.19	99.39	113.58	127.78
71.19	85.43	99.67	113.90	128.14
71.39	85.67	99.95	114.22	128.50
71.59	85.91	100.23	114.54	128.86
71.79	86.15	100.51	114.86	129.22
71.90	86.30	100.79	115.18	129.58

Diam. in Inches.	DEPTH.			
	1	2	3	4
72.0	14.4380	28.88	43.31	57.75
.1	14.4781	28.96	43.43	57.91
.2	14.5183	29.04	43.55	58.07
.3	14.5585	29.12	43.67	58.23
.4	14.5988	29.20	43.80	58.40
.5	14.6392	29.28	43.92	58.56
.6	14.6796	29.36	44.04	58.72
.7	14.7201	29.44	44.16	58.88
.8	14.7606	29.52	44.28	59.04
.9	14.8012	29.60	44.40	59.20
73.0	14.8418	29.68	44.53	59.37
.1	14.8825	29.76	44.65	59.53
.2	14.9232	29.85	44.77	59.69
.3	14.9640	29.93	44.89	59.86
.4	15.0049	30.01	45.02	60.02
.5	15.0458	30.09	45.14	60.18
.6	15.0868	30.17	45.26	60.35
.7	15.1278	30.26	45.38	60.51
.8	15.1689	30.34	45.51	60.68
.9	15.2100	30.42	45.63	60.84
74.0	15.2512	30.50	45.75	61.00
.1	15.2925	30.58	45.88	61.17
.2	15.3338	30.67	46.00	61.34
.3	15.3751	30.75	46.13	61.50
.4	15.4165	30.83	46.25	61.66
.5	15.4580	30.92	46.37	61.83
.6	15.4950	30.99	46.49	61.98
.7	15.5411	31.08	46.62	62.16
.8	15.5827	31.17	46.75	62.33
.9	15.6244	31.25	46.87	62.50

DEPTH.				
5	6	7	8	9
72.19	86.63	101.07	115.50	129.94
72.39	86.87	101.35	115.82	130.30
72.59	87.11	101.63	116.14	130.66
72.79	87.35	101.91	116.46	131.02
73.00	87.59	102.19	116.79	131.39
73.20	87.83	102.47	117.11	131.75
73.40	88.07	102.75	117.43	132.11
73.60	88.32	103.04	117.76	132.48
73.80	88.56	103.32	118.08	132.84
74.01	88.81	103.61	118.41	132.21
74.21	89.05	103.89	118.74	133.58
74.41	89.29	104.17	119.06	133.94
74.62	89.54	104.46	119.38	134.31
74.82	89.78	104.75	119.71	134.68
75.03	90.03	105.04	120.04	135.05
75.23	90.28	105.32	120.37	135.41
75.44	90.52	105.61	120.70	135.78
75.64	90.77	105.90	121.02	136.15
75.85	91.01	106.18	121.35	136.52
76.05	91.26	106.47	121.68	136.89
76.26	91.51	106.76	122.01	137.26
76.46	91.75	107.04	122.34	137.63
76.67	92.00	107.34	122.67	138.01
76.88	92.25	107.63	123.00	138.38
77.08	92.50	107.91	123.33	138.74
77.29	92.75	108.21	123.66	139.12
77.48	92.97	108.47	123.96	139.46
77.71	93.25	108.79	124.33	139.87
77.92	93.50	109.08	124.66	140.25
78.12	93.74	109.37	124.99	140.62

Diam. in Inches.	DEPTH.			
	1	2	3	4
75.0	15.6662	31.33	47.00	62.66
.1	15.7080	31.42	47.12	62.83
.2	15.7499	31.50	47.25	63.00
.3	15.7918	31.58	47.38	63.17
.4	15.8337	31.67	47.50	63.34
.5	15.8758	31.75	47.63	63.50
.6	15.9178	31.84	47.75	63.67
.7	15.9600	31.92	47.88	63.84
.8	16.0022	32.00	48.01	64.01
.9	16.0444	32.09	48.13	64.18
76.0	16.0867	32.17	48.26	64.35
.1	16.1291	32.26	48.39	64.52
.2	16.1715	32.34	48.51	64.68
.3	16.2140	32.43	48.64	64.86
.4	16.2565	32.51	48.77	65.02
.5	16.2991	32.60	48.90	65.20
.6	16.3417	32.68	49.03	65.27
.7	16.3844	32.77	49.15	65.54
.8	16.4272	32.85	49.28	65.71
.9	16.4700	32.94	49.41	65.88
77.0	16.5129	33.03	49.54	66.05
.1	16.5558	33.11	49.67	66.22
.2	16.5988	33.20	49.80	66.40
.3	16.6418	33.28	49.93	66.57
.4	16.6849	33.37	50.06	66.74
.5	16.7280	33.46	50.18	66.91
.6	16.7712	33.54	50.31	67.08
.7	16.8145	33.63	50.44	67.26
.8	16.8578	33.72	50.57	67.43
.9	16.9011	33.80	50.70	67.60

DEPTH.				
5	6	7	8	9
78.33	94.00	109.66	125.33	140.99
78.54	94.25	109.96	125.66	141.37
78.75	94.50	110.25	126.00	141.75
78.96	94.75	110.54	126.34	142.13
79.17	95.00	110.84	126.67	142.51
79.38	95.26	111.13	127.01	142.88
79.59	95.51	111.43	127.34	143.26
79.80	96.76	111.72	127.68	143.64
80.01	96.01	112.01	128.02	144.02
80.22	96.26	112.31	128.35	144.40
80.44	96.52	112.61	128.70	144.78
80.65	96.77	112.90	129.03	145.16
80.86	97.03	113.20	129.37	145.54
81.07	97.28	113.50	129.71	145.93
81.28	97.54	113.79	130.05	146.30
81.50	97.80	114.10	130.40	146.70
81.71	98.05	114.39	130.74	147.08
81.92	98.30	114.69	131.07	147.46
82.14	98.56	114.99	131.42	147.84
82.35	98.82	115.29	131.76	148.23
82.57	99.08	115.59	132.10	148.62
82.78	99.34	115.89	132.45	149.00
83.00	99.59	116.19	132.79	149.39
83.21	99.85	116.49	133.14	149.78
83.43	100.11	116.80	133.48	150.17
83.64	100.37	117.10	133.82	150.55
83.86	100.63	117.40	134.17	150.94
84.07	100.88	117.70	134.51	151.33
84.29	101.15	118.01	134.86	151.72
84.51	101.41	118.31	135.21	152.11

Diam. in Inches.	DEPTH.			
	1	2	3	4
78.0	16.9445	33.89	50.83	67.78
.1	16.9880	33.98	50.96	67.95
.2	17.0316	34.06	51.09	68.12
.3	17.0751	34.15	51.23	68.30
.4	17.1188	34.24	51.36	68.48
.5	17.1625	34.32	51.49	68.65
.6	17.2062	34.41	51.62	68.82
.7	17.2500	34.50	51.75	69.00
.8	17.2939	34.59	51.88	69.18
.9	17.3378	34.68	52.01	69.35
79.0	17.3818	34.76	52.15	69.53
.1	17.4258	34.85	52.28	69.70
.2	17.4699	34.94	52.41	69.88
.3	17.5141	35.03	52.54	70.06
.4	17.5583	35.12	52.67	70.23
.5	17.6025	35.20	52.81	70.41
.6	17.6468	35.29	52.94	70.59
.7	17.6912	35.38	53.07	70.76
.8	17.7356	35.47	53.21	70.94
.9	17.7801	35.56	53.34	71.12
80.0	17.8246	35.65	53.47	71.30
.1	17.8692	35.74	53.61	71.48
.2	17.9139	35.83	53.74	71.66
.3	17.9586	35.92	53.87	71.83
.4	18.0033	36.01	54.01	72.01
.5	18.0481	36.10	54.14	72.19
.6	18.0930	36.19	54.28	72.37
.7	18.1379	36.28	54.41	72.55
.8	18.1829	36.37	54.55	72.73
.9	18.2280	36.46	54.68	72.91

DEPTH.				
5	6	7	8	9
84.72	101.66	118.61	135.55	152.50
84.94	101.93	118.92	135.90	152.89
85.16	102.19	119.22	136.25	153.28
85.38	102.45	119.53	136.60	153.68
85.60	102.71	119.83	136.95	154.07
85.81	102.97	120.13	137.30	154.46
86.03	103.24	120.44	137.65	154.85
86.25	103.50	120.75	138.00	155.25
86.47	103.76	121.06	138.35	155.65
86.69	104.03	121.37	138.70	156.04
86.91	104.29	121.67	139.06	156.44
87.13	104.56	121.98	139.41	156.83
87.35	104.82	122.29	139.76	157.23
87.57	105.08	122.60	140.11	157.63
87.79	105.35	122.91	140.46	158.02
88.01	105.61	123.21	140.82	158.42
88.24	105.88	123.53	141.18	158.82
88.46	106.15	123.84	141.53	159.22
88.68	106.41	124.15	141.88	159.62
88.90	106.68	124.46	142.24	160.02
89.12	106.94	124.77	142.59	160.42
89.35	107.21	125.08	142.95	160.82
89.57	107.48	125.40	143.31	161.23
89.79	107.75	125.71	143.66	161.62
90.02	108.02	126.02	144.03	162.03
90.24	108.29	126.34	144.39	162.43
90.47	108.56	126.65	144.74	162.84
90.69	108.83	126.97	145.10	163.24
90.92	109.10	127.28	145.46	163.65
91.14	109.37	127.60	145.82	164.05

Diam. in Inches.	DEPTH.			
	1	2	3	4
81.0	18.2730	36.55	54.82	73.09
.1	18.3182	36.64	54.95	73.27
.2	18.3634	36.73	55.09	73.45
.3	18.4086	36.82	55.22	73.63
.4	18.4540	36.91	55.36	73.82
.5	18.4993	37.00	55.50	74.00
.6	18.5448	37.09	55.64	74.18
.7	18.5902	37.18	55.77	74.36
.8	18.6358	37.27	55.91	74.54
.9	18.6814	37.36	56.04	74.72
82.0	18.7270	37.45	56.18	74.91
.1	18.7727	37.55	56.32	75.09
.2	18.8185	37.64	56.45	75.27
.3	18.8643	37.73	56.59	75.46
.4	18.9102	37.82	56.73	75.64
.5	18.9561	37.91	56.87	75.82
.6	19.0021	38.00	57.01	76.01
.7	19.0481	38.10	57.14	76.19
.8	19.0942	38.19	57.28	76.38
.9	19.1403	38.28	57.42	76.56
83.0	19.1866	38.37	57.56	76.74
.1	19.2328	38.47	57.70	76.93
.2	19.2791	38.56	57.34	77.12
.3	19.3255	38.65	57.98	77.30
.4	19.3719	38.74	58.12	77.49
.5	19.4184	38.84	58.25	77.67
.6	19.4650	38.93	58.40	77.86
.7	19.5115	39.02	58.53	78.04
.8	19.5582	39.12	58.67	78.23
.9	19.6049	39.21	58.82	78.42

DEPTH.				
5	6	7	8	9
91.37	109.64	127.91	146.18	164.46
91.59	109.91	128.23	146.55	164.86
91.82	110.18	128.54	146.91	165.27
92.04	110.45	128.86	147.27	165.67
92.27	110.72	129.18	147.63	166.09
92.50	110.99	129.49	147.99	166.49
92.73	111.27	129.82	148.36	166.91
92.95	111.54	130.13	148.72	167.31
93.18	111.82	130.45	149.09	167.72
93.41	112.09	130.77	149.45	168.13
93.64	112.36	131.09	149.81	168.54
93.87	112.64	131.41	150.18	168.96
94.09	112.91	131.73	150.54	169.36
94.32	113.18	132.05	150.91	169.78
94.55	113.46	132.37	151.28	170.19
94.78	113.74	132.69	151.65	170.60
95.01	114.01	133.01	152.02	171.02
95.24	114.29	133.34	152.39	171.43
95.47	114.56	133.66	152.75	271.85
95.70	114.84	133.98	153.12	172.26
95.93	115.12	134.30	153.49	172.67
96.17	115.40	134.63	153.86	173.10
96.40	115.67	134.95	154.23	173.51
96.63	115.95	135.28	154.60	173.93
96.86	116.23	135.60	154.98	174.35
97.09	116.51	135.93	155.34	174.76
97.33	116.79	136.26	155.72	175.19
97.56	117.07	136.58	156.09	175.60
97.79	117.35	136.91	156.46	176.02
98.03	117.63	137.24	156.84	176.45

Diam. in Inches.	DEPTH.			
	1	2	3	4
84.0	19.6517	39.30	58.96	78.61
.1	19.6985	39.40	59.09	78.79
.2	19.7454	39.49	59.24	78.98
.3	19.7923	39.58	59.38	79.17
.4	19.8393	39.68	59.52	79.36
.5	19.8863	39.77	59.66	79.54
.6	19.9334	39.87	59.80	79.73
.7	19.9806	39.96	59.94	79.92
.8	20.0278	40.06	60.08	80.11
.9	20.0750	40.15	60.23	80.30
85.0	20.1223	40.24	60.37	80.49
.1	20.1697	40.34	60.51	80.68
.2	20.2172	40.43	60.65	80.87
.3	20.2646	40.53	60.79	81.06
.4	20.3122	40.62	60.94	81.25
.5	20.3598	40.72	61.08	81.44
.6	20.4074	40.81	61.22	81.63
.7	20.4551	40.91	61.37	81.82
.8	20.5029	41.01	61.51	82.01
.9	20.5507	41.10	61.65	82.20
86.0	20.5986	41.20	61.79	82.39
.1	20.6465	41.29	61.94	82.58
.2	20.6945	41.39	62.08	82.78
.3	20.7426	41.48	62.23	82.97
.4	20.7907	41.58	62.37	83.16
.5	20.8388	41.68	62.52	83.36
.6	20.8870	41.77	62.66	83.55
.7	20.9353	41.87	62.81	83.74
.8	20.9836	41.97	62.95	83.93
.9	21.0320	42.06	63.10	84.13

DEPTH.				
5	6	7	8	9
98.26	117.91	137.56	157.22	176.87
98.49	118.19	137.89	157.58	177.28
98.73	118.47	138.22	157.96	177.71
98.96	118.75	138.54	158.34	178.13
99.20	119.03	138.87	158.71	178.55
99.43	119.32	139.20	159.09	178.97
99.67	119.60	139.53	159.46	179.40
99.90	119.88	139.86	159.84	179.82
100.14	120.17	140.20	160.22	180.25
100.38	120.45	140.53	160.60	180.68
100.61	120.73	140.85	160.98	181.10
100.86	121.02	141.19	161.36	181.53
101.09	121.30	141.52	161.74	181.95
101.32	121.58	141.85	162.11	182.38
101.56	121.87	142.18	162.50	182.81
101.80	122.16	142.52	162.88	183.24
102.04	122.44	142.85	163.26	183.66
102.28	122.73	143.19	163.64	184.10
102.52	123.02	143.52	164.02	184.53
102.76	123.31	143.86	164.41	184.96
102.99	123.59	144.19	164.78	185.38
103.23	123.88	144.52	165.17	185.82
103.47	124.16	144.86	165.55	186.25
103.71	124.45	145.19	165.94	186.68
103.96	124.75	145.54	166.33	187.12
104.20	125.03	145.87	166.71	187.55
104.44	125.32	146.21	167.10	187.98
104.68	125.61	146.55	167.48	188.42
104.92	125.90	146.88	167.86	188.85
105.16	126.19	147.22	168.26	189.29

Diam. in Inches.	DEPTH.			
	1	2	3	4
87.0	21.0804	42.16	63.24	84.32
.1	21.1289	42.26	63.39	84.52
.2	21.1775	42.35	63.53	84.71
.3	21.2261	42.45	63.68	84.90
.4	21.2747	42.55	63.83	85.10
.5	21.3234	42.65	63.97	85.29
.6	21.3722	42.74	64.12	85.49
.7	21.4210	42.84	64.26	85.68
.8	21.4699	42.94	64.41	85.88
.9	21.5188	43.04	64.56	86.08
88.0	21.5678	43.14	64.70	86.27
.1	21.6169	43.23	64.85	86.47
.2	21.6660	43.33	65.00	86.66
.3	21.7151	43.43	65.15	86.86
.4	21.7643	43.53	65.29	87.06
.5	21.8136	43.63	65.44	87.25
.6	21.8629	43.73	65.59	87.45
.7	21.9123	43.82	65.74	87.65
.8	21.9617	43.92	65.89	87.85
.9	22.0112	44.02	66.03	88.04
89.0	22.0608	44.12	66.18	88.24
.1	22.1104	44.22	66.33	88.44
.2	22.1600	44.32	66.48	88.64
.3	22.2098	44.42	66.63	88.84
.4	22.2595	44.52	66.78	89.04
.5	22.3093	44.62	66.93	89.24
.6	22.3592	44.72	67.08	89.44
.7	22.4092	44.82	67.23	89.64
.8	22.4592	44.92	67.38	89.84
.9	22.5092	45.02	67.53	90.04

DEPTH.				
5	6	7	8	9
105.40	126.48	147.56	168.64	189.72
105.65	126.77	147.90	169.03	190.16
105.89	127.06	148.24	169.42	190.59
106.13	127.36	148.58	169.81	191.03
106.38	127.65	148.93	170.20	191.48
106.62	127.94	149.26	170.58	191.91
106.86	128.23	149.60	170.98	192.35
107.11	128.53	149.95	171.37	192.79
107.35	128.82	150.29	171.76	193.23
107.60	129.11	150.63	172.15	193.67
107.84	129.41	150.98	172.54	194.91
108.09	129.70	151.32	172.94	194.35
108.33	130.00	151.66	173.33	194.79
108.58	130.29	152.01	173.72	195.23
108.82	130.58	152.35	174.11	195.67
109.07	130.88	152.69	174.50	196.32
109.32	131.18	153.04	174.90	196.77
109.56	131.47	153.38	175.30	197.21
109.81	131.77	153.73	175.70	197.66
110.06	132.07	154.08	176.09	198.10
110.31	132.37	154.43	176.49	198.55
110.55	132.66	154.77	176.88	198.99
110.80	132.96	155.12	177.28	199.44
111.05	133.26	155.47	177.68	199.89
111.30	133.55	155.81	178.07	200.33
111.55	133.85	156.16	178.47	200.78
111.80	134.15	156.51	178.87	201.23
112.05	134.45	156.86	179.27	201.68
112.30	134.75	157.21	179.67	202.13
112.55	135.05	157.56	180.07	202.58

Diam. in Inches.	DEPTH.			
	1	2	3	4
90.0	22.5593	45.12	67.68	90.24
.1	22.6095	45.22	67.83	90.44
.2	22.6597	45.32	67.98	90.64
.3	22.7100	45.42	68.13	90.84
.4	22.7603	45.52	68.28	91.04
.5	22.8107	45.62	68.43	91.24
.6	22.8611	45.72	68.58	91.44
.7	22.9116	45.82	68.73	91.64
.8	22.9621	45.92	68.89	91.85
.9	23.0128	46.03	69.04	92.05
91.0	23.0634	46.13	69.19	92.25
.1	23.1141	46.23	69.34	92.46
.2	23.1649	46.33	69.50	92.66
.3	23.2157	46.43	69.65	92.86
.4	23.2666	46.53	69.80	93.06
.5	23.3176	46.63	69.95	93.27
.6	23.3685	46.74	70.10	93.47
.7	23.4196	46.84	70.26	93.68
.8	23.4707	46.94	70.41	93.88
.9	23.5219	47.04	70.57	94.09
92.0	23.5731	47.15	70.72	94.29
.1	23.6244	47.25	70.87	94.50
.2	23.6757	47.35	71.03	94.70
.3	23.7271	47.45	71.18	94.91
.4	23.7785	47.56	71.33	95.11
.5	23.8300	47.66	71.49	95.32
.6	23.8816	47.76	71.64	95.52
.7	23.9332	47.87	71.80	95.73
.8	23.9848	47.97	71.96	95.94
.9	24.0366	48.07	72.11	96.14

DEPTH.				
5	6	7	8	9
112.80	135.35	157.91	180.47	203.03
113.05	135.65	158.26	180.87	203.48
113.30	135.96	158.62	181.28	203.94
113.55	136.26	158.97	181.68	204.39
113.80	136.56	159.32	182.08	204.84
114.05	136.86	159.67	182.48	205.29
114.31	137.17	160.03	182.89	205.75
114.56	137.47	160.38	183.29	206.20
114.81	137.77	160.73	183.70	206.66
115.07	138.08	161.09	184.10	206.12
115.32	138.38	161.44	184.50	207.57
115.57	138.68	161.80	184.91	208.03
115.83	138.99	162.16	185.32	208.49
116.08	139.30	162.51	185.73	208.94
116.33	139.60	162.86	186.13	209.39
116.59	139.90	163.22	186.54	209.85
116.84	140.21	163.58	186.94	210.31
117.10	140.51	163.93	187.35	210.77
117.36	140.83	164.30	187.77	211.24
117.61	141.13	164.65	188.18	211.70
117.87	141.44	165.01	188.58	212.16
118.12	141.74	165.37	188.99	212.62
118.38	142.05	165.73	189.40	213.08
118.64	142.36	166.09	189.82	213.54
118.89	142.67	166.45	190.22	214.00
119.15	142.98	166.81	190.64	214.47
119.41	143.29	167.17	191.05	214.93
119.67	143.60	167.53	191.46	210.40
119.93	143.91	167.90	191.88	215.87
120.18	144.22	168.25	192.29	216.32

Diam. in Inches.	DEPTH.			
	1	2	3	4
93.0	24.0883	48.18	72.26	96.35
.1	24.1402	48.28	72.42	96.56
.2	24.1920	48.38	72.58	96.77
.3	24.2440	48.49	72.73	96.98
.4	24.2960	48.59	72.89	97.18
.5	24.3480	48.70	73.04	97.39
.6	24.4001	48.80	73.20	97.60
.7	24.4523	48.90	73.36	97.81
.8	24.5045	49.01	73.51	98.02
.9	24.5568	49.11	73.67	98.23
94.0	24.6091	49.22	73.83	98.44
.1	24.6615	49.32	73.98	98.64
.2	24.7140	49.43	74.14	98.86
.3	24.7665	49.53	74.30	99.06
.4	24.8190	49.64	74.46	99.28
.5	24.8716	49.74	74.61	99.48
.6	24.9243	49.85	74.77	99.70
.7	24.9770	49.95	74.93	99.91
.8	25.0298	50.06	75.09	100.12
.9	25.0826	50.16	75.25	100.33
95.0	25.1355	50.27	75.41	100.54
.1	25.1885	50.38	75.56	100.75
.2	25.2415	50.48	75.72	100.96
.3	25.2945	50.59	75.88	101.18
.4	25.3476	50.69	76.04	101.39
.5	25.4008	50.80	76.20	101.60
.6	25.4540	50.91	76.36	101.82
.7	25.5073	51.01	76.52	102.03
.8	25.5606	51.12	76.68	102.24
.9	25.6140	51.23	76.84	102.46

DEPTH.				
5	6	7	8	9
120.44	144.53	168.61	192.70	216.79
120.70	144.84	168.98	193.12	217.26
120.96	145.15	169.34	193.54	217.73
121.22	145.46	169.70	193.95	218.20
121.48	145.78	170.07	194.37	218.66
121.74	146.09	170.43	194.78	219.13
122.00	146.40	170.80	195.20	219.60
122.26	146.71	171.16	195.62	220.07
122.52	147.02	171.52	196.03	220.54
122.79	147.34	171.89	196.46	220.01
123.05	147.65	172.26	196.87	221.48
123.31	147.97	172.62	197.29	221.95
123.57	148.28	173.00	197.71	222.43
123.83	148.60	173.36	198.13	222.89
124.10	148.91	173.73	198.55	223.37
124.36	149.23	174.09	198.97	223.84
124.62	149.54	174.46	199.39	224.32
124.89	149.86	174.83	199.82	224.79
125.15	150.18	175.21	200.24	225.27
125.41	150.49	175.57	200.66	225.74
125.68	150.81	175.94	201.08	226.22
125.94	151.13	176.31	201.50	226.69
126.21	151.45	176.68	201.93	227.17
126.47	151.76	177.05	202.35	227.65
126.74	152.08	177.42	202.78	228.12
127.01	152.41	177.80	203.21	228.61
127.27	152.72	178.17	203.63	229.09
127.54	153.04	178.54	204.06	229.56
127.80	153.36	178.92	204.48	230.04
128.07	153.68	179.29	204.91	230.53

Diam. in Inches.	DEPTH.			
	1	2	3	4
96.0	25.6675	51.33	77.00	102.67
.1	25.7210	51.44	27.16	102.88
.2	25.7745	51.55	77.32	103.10
.3	25.8282	51.66	77.48	103.31
.4	25.8818	51.76	77.65	103.53
.5	25.9355	51.87	77.81	103.74
.6	25.9893	51.98	77.97	103.96
.7	26.0432	52.09	78.13	104.17
.8	26.0971	52.19	78.29	104.39
.9	26.1510	52.30	78.45	104.60
97.0	26.2050	52.41	78.62	104.82
.1	26.2591	52.52	78.78	105.04
.2	26.3132	52.63	78.94	105.25
.3	26.3673	52.73	79.10	105.47
.4	26.4216	52.84	79.26	105.68
.5	26.4759	52.95	79.43	105.90
.6	26.5302	53.06	79.59	106.12
.7	26.5846	53.17	79.75	106.34
.8	26.6390	53.28	79.92	106.56
.9	26.6935	53.39	80.08	106.77
98.0	26.7481	53.50	80.24	106.99
.1	26.8027	53.61	80.41	107.21
.2	26.8574	53.71	80.57	107.43
.3	26.9121	53.82	80.74	107.65
.4	26.9669	53.93	80.90	107.87
.5	27.0217	54.04	81.07	108.09
.6	27.0766	54.15	81.23	108.30
.7	27.1316	54.26	81.39	108.52
.8	27.1866	54.37	81.56	108.74
.9	27.2416	54.48	81.72	108.96

DEPTH.				
5	6	7	8	9
128.34	154.00	179.67	205.34	231.00
128.61	154.33	180.05	205.77	231.49
128.87	154.64	180.42	206.19	231.97
129.14	154.97	180.80	206.62	232.45
129.41	155.29	181.17	207.06	232.94
129.68	155.61	181.55	207.48	233.42
129.95	155.93	181.92	207.91	233.90
130.22	156.26	182.30	208.34	234.39
130.49	156.58	182.68	208.78	234.87
130.76	156.91	183.06	209.21	235.36
131.03	157.23	183.44	209.64	235.85
131.30	157.55	183.81	210.07	236.33
131.57	157.88	184.19	210.50	236.82
131.84	158.20	184.57	210.94	237.30
132.11	158.53	184.95	211.37	237.79
132.38	158.86	185.33	211.81	238.28
132.65	159.18	185.71	212.24	238.77
132.92	159.50	186.09	212.67	239.26
133.20	159.83	186.47	213.11	239.75
133.46	160.16	186.85	213.54	240.24
133.74	160.49	187.24	213.98	240.73
134.02	160.82	187.62	214.42	241.23
134.29	161.14	188.00	214.86	241.71
134.56	161.47	188.38	215.30	242.21
134.84	161.80	188.77	215.74	242.70
135.11	162.13	189.15	216.18	243.20
135.38	162.46	189.53	216.61	243.68
135.66	162.79	189.92	217.05	244.18
135.93	163.12	190.30	217.49	244.67
136.21	163.45	190.69	217.93	245.17

Diam. in Inches.	DEPTH. 1	2	3	4
99.0	27.2968	54.59	81.89	109.19
.1	27.3519	54.70	82.06	109.41
.2	27.4072	54.81	82.22	109.63
.3	27.4625	54.92	82.39	109.85
.4	27.5178	55.04	82.55	110.07
.5	27.5732	55.15	82.72	110.29
.6	27.6286	55.26	82.88	110.51
.7	27.6841	55.37	83.05	110.74
.8	27.7397	55.48	83.22	110.96
.9	27.7953	55.59	83.39	111.18
100.0	27.8510	55.70	83.55	111.40
.1	27.9067	55.81	83.72	111.63
.2	27.9625	55.92	83.89	111.85
.3	28.0184	56.04	84.05	112.07
.4	28.0743	56.15	84.22	112.30
.5	28.1302	56.26	84.39	112.51
.6	28.1862	56.37	84.56	112.74
.7	28.2423	56.48	84.73	112.97
.8	28.2984	56.60	84.89	113.19
.9	28.3546	56.71	85.06	113.42
101.0	28.4108	59.82	85.23	113.64
.1	28.4671	56.93	85.40	113.87
.2	28.5234	57.05	85.57	114.09
.3	28.5798	57.16	85.74	114.32
.4	28.6363	57.27	85.91	114.54
.5	28.6928	57.39	86.08	114.77
.6	28.7494	57.50	86.25	115.00
.7	28.8060	57.61	86.42	115.22
.8	28.8627	57.73	86.59	115.45
.9	28.9194	57.84	86.76	115.68

DEPTH.				
5	6	7	8	9
136.49	163.78	191.08	218.38	245.67
136.76	164.11	191.46	218.82	246.17
137.04	164.44	191.85	219.26	246.66
137.31	164.77	192.23	219.70	247.16
137.59	165.11	192.63	220.14	247.66
137.87	165.44	193.01	220.58	248.16
138.14	165.77	193.40	221.02	248.65
138.42	166.10	193.79	221.47	249.16
138.70	166.44	194.18	221.92	249.66
138.98	166.77	194.57	222.36	250.16
139.26	167.11	194.96	222.81	250.66
139.54	167.44	195.35	223.26	251.16
139.81	167.77	195.73	223.70	251.66
140.09	168.11	196.13	224.14	252.16
140.37	168.44	196.52	224.59	252.67
140.65	168.78	196.91	225.04	253.17
140.93	169.12	197.30	225.49	253.67
141.21	169.45	197.69	225.94	254.18
141.49	169.79	198.09	226.38	254.68
141.77	170.12	198.48	226.83	255.19
142.06	170.47	198.88	227.29	255.70
142.34	170.80	199.27	227.74	256.20
142.62	171.14	199.66	228.18	256.71
142.90	171.48	200.06	228.64	257.22
143.18	171.82	200.45	229.09	257.72
143.47	172.16	200.85	229.54	258.24
143.75	172.49	201.24	229.99	258.74
144.03	172.84	201.64	230.45	259.25
144.32	173.18	202.04	230.90	259.77
144.60	173.51	202.43	231.35	260.27

Diam. in Inches.	DEPTH.			
	1	2	3	4
102.0	28.9762	57.95	87.93	115.90
.1	29.0330	58.07	87.10	116.13
.2	29.0899	58.18	87.27	116.36
.3	29.1469	58.29	87.44	116.59
.4	29.2039	58.41	87.61	116.82
.5	29.2610	58.52	87.78	117.04
.6	29.3181	58.64	87.95	117.27
.7	29.3753	58.75	88.13	117.50
.8	29.4325	58.86	88.30	117.73
.9	29.4898	58.98	88.47	117.96
103.0	29.5471	59.09	88.64	118.19
.1	29.6045	59.21	88.81	118.41
.2	29.6620	59.32	88.99	118.65
.3	29.7195	59.44	89.16	118.88
.4	29.7771	59.55	89.33	119.11
.5	29.8347	59.67	89.51	119.34
.6	29.8924	59.78	89.68	119.57
.7	29.9501	59.90	89.85	119.80
.8	30.0079	60.02	90.02	120.03
.9	30.0657	60.13	90.20	120.26
104.0	30.1236	60.25	90.37	120.49
.1	30.1816	60.36	90.54	120.72
.2	30.2396	60.48	90.72	120.96
.3	30.2977	60.60	90.89	121.19
.4	30.3558	60.71	91.07	121.41
.5	30.4140	60.81	91.24	121.66
.6	30.4722	60.94	91.42	121.89
.7	30.5305	61.06	91.59	122.12
.8	30.5889	61.18	91.77	122.36
.9	30.6473	61.29	91.94	122.59

DEPTH.				
5	6	7	8	9
144.88	173.86	202.83	231.81	261.78
145.17	174.20	203.23	232.26	261.30
145.45	174.54	203.63	232.72	261.81
145.74	174.88	204.03	233.18	262.32
146.02	175.22	204.43	233.63	262.84
146.31	175.57	204.83	234.09	263.35
146.59	175.91	205.23	234.54	263.86
147.88	176.25	205.63	235.00	264.38
147.16	176.59	206.02	235.46	264.89
147.45	176.94	206.43	235.92	265.41
147.74	177.28	206.83	236.38	265.92
148.02	177.62	207.23	236.83	266.44
148.31	177.97	207.63	237.30	266.96
148.66	178.31	208.03	237.75	267.47
148.89	178.66	208.44	238.22	267.99
149.18	179.01	208.85	238.68	268.52
149.46	179.35	209.24	239.14	269.03
149.76	179.70	209.65	239.60	269.55
150.04	180.05	210.06	240.06	270.07
150.33	180.40	210.46	240.53	270.59
150.62	180.74	210.86	249.98	271.11
150.91	181.09	211.27	241.45	271.63
151.20	181.43	211.67	241.91	272.15
151.49	181.79	212.09	242.38	272.68
151.78	182.14	212.49	242.85	273.20
152.07	182.48	212.90	243.31	273.73
152.36	182.83	213.30	243.78	274.25
152.65	183.18	213.71	244.24	274.77
152.95	183.53	214.12	244.71	275.30
153.24	183.88	214.53	245.18	275.82

Diam. in Inches.	DEPTH.			
	1	2	3	4
105.0	30.7057	61.41	92.12	122.82
.1	30.7642	61.53	92.29	123.06
.2	30.8228	61.65	92.47	123.29
.3	30.8814	61.76	92.64	123.52
.4	30.9401	61.88	92.82	123.76
.5	30.9989	62.00	93.00	124.00
.6	31.0577	62.12	93.17	124.23
.7	31.1165	62.23	93.35	124.46
.8	31.1754	62.35	93.53	124.70
.9	31.2344	62.47	93.70	124.94
106.0	31.2934	62.59	93.88	125.17
.1	31.3525	62.70	94.06	125.41
.2	31.4116	62.82	94.23	125.64
.3	31.4708	62.94	94.41	125.88
.4	31.5300	63.06	94.59	126.12
.5	31.5893	63.18	94.77	126.36
.6	31.6487	63.30	94.95	126.60
.7	31.7081	63.42	95.12	126.83
.8	31.7675	63.53	95.30	127.07
.9	31.8270	63.65	95.48	127.30
107.0	31.8866	63.77	95.66	127.54
.1	31.9462	63.89	95.84	127.78
.2	32.0059	64.01	96.02	128.02
.3	32.0657	64.13	96.20	128.26
.4	32.1255	64.25	96.38	128.50
.5	32.1853	64.37	96.56	128.74
.6	32.2452	64.49	96.74	128.98
.7	32.3052	64.61	96.92	129.22
.8	32.3652	64.73	97.10	129.46
.9	32.4252	64.85	97.28	129.70

DEPTH.				
5	6	7	8	9
153.53	184.24	214.94	24 .65	276.35
153.82	184.58	215.35	246.11	276.88
154.12	184.94	215.76	246.58	277.41
154.41	185.29	216.17	247.05	277.93
154.70	185.64	216.58	247.52	278.46
155.00	185.99	216.99	247.99	278.99
155.29	186.35	217.41	248.46	279.52
155.58	186.70	217.81	248.93	280.04
155.88	187.05	218.23	249.40	280.58
156.17	187.40	218.64	249.87	281.11
156.47	187.76	219.05	250.34	281.64
156.76	188.11	219.46	250.82	282.17
157.06	188.47	219.88	251.29	282.70
157.36	188.83	220.30	251.77	283.24
157.65	189.18	220.71	252.24	283.77
157.95	189.53	221.12	252.71	284.30
158.25	189.89	221.54	253.19	284.84
158.54	190.25	221.96	253.66	285.37
158.84	190.60	222.37	254.14	285.90
159.14	190.96	222.79	254.62	286.44
159.43	191.32	223.20	255.09	286.97
159.73	191.68	223.62	255.57	287.51
160.03	192.04	224.04	256.05	288.05
160.33	192.40	224.46	256.53	288.59
160.63	192.75	224.88	257.00	289.13
160.93	193.11	225.30	257.48	289.67
161.23	193.47	225.72	257.96	290.21
161.53	193.83	226.14	258.44	290.75
161.83	194.19	226.56	258.92	291.29
162.13	194.55	226.98	259.40	291.83

Diam. in Inches.	DEPTH.			
	1	2	3	4
108.0	32.4854	64.97	97.46	129.94
.1	32.5456	65.09	97.64	130.18
.2	32.6058	65.21	97.82	130.42
.3	32.6661	65.33	98.00	130.66
.4	32.7265	65.45	98.18	130.90
.5	32.7869	65.57	98.36	131.15
.6	32.8474	65.69	98.54	131.39
.7	32.9079	65.82	98.72	131.63
.8	32.9685	65.94	98.90	131.87
.9	33.0291	66.06	99.09	132.12
109.0	33.0898	66.18	99.27	132.36
.1	33.1505	66.30	99.45	132.60
.2	33.2113	66.42	99.63	132.84
.3	33.2722	66.54	99.82	133.09
.4	33.3331	66.67	100.00	133.33
.5	33.3940	66.79	100.18	133.58
.6	33.4551	66.91	100.37	133.82
.7	33.5161	67.03	100.55	134.06
.8	33.5773	67.16	100.73	134.31
.9	33.6385	67.28	100.92	134.56
110.0	33.6997	67.40	101.10	134.80
.1	33.7610	67.52	101.28	135.04
.2	33.8224	67.64	101.47	135.29
.3	33.8838	67.77	101.65	135.54
.4	33.9452	67.89	101.84	135.78
.5	34.0068	68.01	102.02	136.03
.6	34.0683	68.14	102.20	136.27
.7	34.1300	68.26	102.39	136.52
.8	34.1917	68.38	102.58	136.77
.9	34.2534	68.51	102.76	137.01

DEPTH.				
5	6	7	8	9
162.43	194.91	227.40	259.88	292.37
162.73	195.27	227.82	260.36	292.91
163.03	195.63	228.24	260.85	293.45
163.33	196.00	228.66	261.33	293.99
163.63	196.36	229.08	261.81	294.53
163.94	196.72	229.51	262.30	295.08
164.24	197.08	229.93	262.78	295.62
164.54	197.45	230.36	263.26	296.17
164.84	197.80	230.78	263.74	296.71
165.15	198.17	231.20	264.23	297.26
165.45	198.54	231.63	264.72	297.81
165.75	198.90	232.05	265.20	298.35
166.06	199.27	232.48	265.69	298.90
166.36	199.63	232.90	266.18	299.45
166.67	200.00	233.33	266.66	300.00
166.97	200.36	233.76	267.15	300.55
167.28	200.73	234.19	267.64	301.10
167.58	201.10	234.61	268.13	301.64
167.89	201.47	235.05	268.62	302.20
168.20	201.83	235.47	269.11	302.75
168.50	202.20	235.90	269.60	303.30
168.81	202.57	236.33	270.09	303.85
169.11	202.93	236.75	270.58	304.40
169.42	203.30	237.19	271.07	304.96
169.73	203.67	237.62	271.56	305.51
170.04	204.04	238.05	272.06	306.06
170.34	204.41	238.48	272.54	306.61
170.65	204.78	238.91	273.04	307.17
170.96	205.15	239.34	273.54	307.73
171.07	205.52	239.77	274.02	308.28

Diam. in Inches.	DEPTH.			
	1	2	3	4
111.0	34.3152	68.63	102.95	137.26
.1	34.3771	68.75	103.13	137.51
.2	34.4390	68.88	103.32	137.76
.3	34.5010	69.00	103.50	138.00
.4	34.5630	69.13	103.69	138.25
.5	34.6251	69.25	103.88	138.50
.6	34.6872	69.37	104.06	138.75
.7	34.7494	69.50	104.25	139.00
.8	34.8116	69.62	104.43	139.24
.9	34.3739	69.75	104.62	139.50
112.0	34.9363	69.87	104.81	139.74
.1	34.9987	70.00	105.00	140.00
.2	35.0612	70.12	105.18	140.24
.3	35.1237	70.25	105.37	140.50
.4	35.1863	70.37	105.56	140.74
.5	35.2489	70.50	105.75	141.00
.6	35.3116	70.62	105.93	141.24
.7	35.3744	70.75	106.12	141.50
.8	35.4372	70.87	106.31	141.75
.9	35.5000	71.00	106.50	142.00
113.0	35.5629	71.13	106.69	142.25
.1	35.6259	71.25	106.88	142.50
.2	35.6889	71.38	107.07	142.76
.3	35.7520	71.50	107.26	143.01
.4	35.8152	71.63	107.45	143.26
.5	35.8784	71.76	107.64	143.51
.6	35.9416	71.88	107.82	143.76
.7	36.0049	72.01	108.02	144.02
.8	36.0683	72.14	108.21	144.27
.9	36.1317	72.26	108.40	144.53

DEPTH.				
5	6	7	8	9
171.58	205.89	240.21	274.52	308.84
171.89	206.26	240.64	275.02	309.39
172.20	206.63	241.07	275.51	309.95
172.51	207.01	241.51	276.01	310.51
172.82	207.38	241.94	276.50	311.07
173.13	207.75	242.38	277.00	311.63
173.44	208.12	242.81	277.50	312.18
173.75	208.49	243.24	277.99	312.74
174.06	208.87	243.68	278.49	313.30
174.37	209.24	244.12	278.99	313.87
174.68	209.62	244.55	279.49	314.42
175.00	209.99	244.99	279.99	314.99
175.31	210.37	245.43	280.49	315.55
175.62	210.74	245.87	280.99	316.12
175.93	211.12	246.30	281.49	316.67
176.25	211.49	246.84	281.99	317.24
176.56	211.87	247.18	282.49	317.80
176.87	212.24	247.62	282.99	318.37
177.19	212.62	248.06	283.50	318.93
177.50	213.00	248.50	284.00	319.50
177.82	213.38	248.94	284.50	320.07
178.13	213.76	249.38	285.01	320.63
178.45	214.13	249.82	285.51	321.20
178.76	214.51	250.26	286.02	321.77
178.08	214.89	250.71	286.52	322.34
179.39	215.27	251.15	287.02	322.90
179.71	215.65	251.59	287.53	323.47
180.03	216.03	252.04	288.04	324.05
180.34	216.41	252.48	288.54	324.61
180.66	216.79	252.92	289.06	325.19

Diam. in Inches.	DEPTH.			
	1	2	3	4
114.0	36.1952	72.39	108.59	144.78
.1	36.2587	72.52	108.78	145.04
.2	36.3223	72.64	108.97	145.29
.3	36.3859	72.77	109.16	145.54
.4	36.4496	72.90	109.35	145.80
.5	36.5134	73.03	109.54	146.05
.6	36.5772	73.15	109.73	146.31
.7	36.6410	73.28	109.92	146.56
.8	36.7049	73.41	110.12	146.82
.9	36.7689	73.54	110.31	147.08
115.0	36.8329	73.67	110.50	147.33
.1	36.8970	73.79	110.69	147.59
.2	36.9612	73.92	110.88	147.84
.3	37.0254	74.05	111.08	148.10
.4	37.0896	74.18	111.27	148.36
.5	37.1539	74.31	111.46	148.62
.6	37.2183	74.44	111.66	148.87
.7	37.2827	74.57	111.85	149.13
.8	37.3472	74.69	112.04	149.39
.9	37.4117	74.82	112.24	149.65
116.0	37.4763	74.95	112.43	149.91
.1	37.5409	75.08	112.62	150.16
.2	37.6056	75.21	112.82	150.42
.3	37.6704	75.34	113.01	150.68
.4	37.7352	75.47	113.21	150.94
.5	37.8001	75.60	113.40	151.20
.6	37.8650	75.73	113.60	151.46
.7	37.9300	75.86	113.79	151.72
.8	37.9950	75.99	113.99	151.98
.9	38.0601	76.12	114.18	152.24

DEPTH.				
5	6	7	8	9
180.98	217.17	253.36	289.56	325.76
181.30	217.55	253.81	290.07	326.33
181.61	217.93	254.25	290.58	326.90
181.93	218.32	254.70	291.09	327.47
182.25	218.69	255.14	291.59	328.04
182.57	218.08	255.59	292.10	328.62
182.89	219.46	256.04	292.62	329.19
183.21	219.85	256.49	293.13	329.77
183.53	220.23	256.94	293.64	330.35
183.85	220.61	257.38	294.15	330.92
184.17	221.00	257.83	294.66	331.50
184.49	221.38	258.28	295.18	332.07
184.81	221.77	258.73	295.69	332.65
185.13	222.15	259.18	296.20	333.23
185.45	222.53	259.62	296.71	333.80
185.78	222.92	260.08	297.23	334.39
186.09	223.31	260.53	297.74	334.96
186.42	223.70	260.98	298.27	335.55
186.74	224.08	261.43	298.78	336.12
187.06	224 47	261.88	299.30	336.71
187.38	224.86	262.33	299.81	337.29
187.71	225.25	262.79	300.33	337.87
188.03	225.63	263.24	300.84	338.45
188.35	226.02	263.69	301.36	339.03
188.68	226.41	264.15	301.88	339.62
189.00	226.80	264.60	302.40	340.20
189.33	227.19	265.06	302.92	340.79
189.65	227.58	265.51	303.44	341.37
189.98	227.97	265.97	303.96	341.96
190.30	228.36	266.42	304.48	342.54

Diam. in Inches.	DEPTH.			
	1	2	3	4
117.0	38.1252	76.25	114.38	152.50
.1	38.1904	76.38	114.57	152.76
.2	38.2557	76.51	114.77	153.02
.3	38.3210	76.64	114.96	153.28
.4	38.3864	76.77	115.16	153.54
.5	38.4518	76.90	115.36	153.81
.6	38.5173	77.03	115.55	154.07
.7	38.5828	77.17	115.75	154.33
.8	38.6484	77.30	115.95	154.59
.9	38.7140	77.43	116.14	154.86
118.0	38.7797	77.56	116.34	155.12
.1	38.8455	77.69	116.54	155.38
.2	38.9113	77.82	116.73	155.64
.3	38.9772	77.95	116.93	155.91
.4	39.0431	78.09	117.13	156.17
.5	39.1091	78.22	117.33	156.44
.6	39.1751	78.35	117.53	156.70
.7	39.2412	78.48	117.72	156.96
.8	39.3073	78.61	117.92	157.23
.9	39.3735	78.75	118.12	157.49
119.0	39.4398	78.88	118.32	157.76
.1	39.5061	79.01	118.52	158.02
.2	39.5725	79.14	118.72	158.29
.3	39.6389	79.28	118.92	158.56
.4	39.7054	79.41	119.12	158.82
.5	39.7719	79.54	119.32	159.09
.6	39.8385	79.68	119.51	159.35
.7	39.9052	79.81	119.72	159.62
.8	39.9719	79.94	119.92	159.89
.9	40.0386	80.08	120.11	160.15

DEPTH.				
5	6	7	8	9
190.63	228.75	266.88	305.00	343.13
190.95	229.14	267.33	305.52	343.71
191.28	229.54	267.79	306.05	344.30
191.61	229.93	268.25	306.57	344.89
191.93	230.32	268.70	307.09	345.47
192.26	230.71	269.16	307.62	346.07
192.59	231.10	269.62	308.14	346.65
192.92	231.50	270.08	308.66	347.25
193.24	231.89	270.54	309.18	347.83
193.57	232.28	271.00	309.71	348.43
193.90	232.68	271.46	310.24	349.02
194.23	233.07	271.92	310.76	349.61
194.56	233.47	272.38	311.29	350.20
194.89	233.86	272.84	311.82	350.79
195.22	234.26	273.30	312.34	351.39
195.55	234.65	273.76	312.87	351.98
195.83	235.06	274.23	313.40	352.58
196.21	235.45	274.69	313.93	353.17
196.54	235.84	275.15	314.46	353.76
196.87	236.24	275.61	314.98	354.36
197.20	236.64	276.08	315.52	354.96
197.53	237.04	276.54	316.05	355.55
197.86	237.43	277.01	316.58	356.15
198.20	237.83	277.47	317.11	356.75
198.53	238.23	277.94	317.64	357.35
198.86	238.63	278.40	318.18	357.95
199.19	239.03	278.87	318.70	358.54
199.53	239.43	279.34	319.24	359.15
199.86	239.83	279.80	319.78	359.75
200.19	240.23	280.27	320.30	365.34

Diam. in Inches.	DEPTH.			
	1	2	3	4
120.0	40.1054	80.21	120.32	160.42
.1	40.1723	80.34	120.52	160.69
.2	40.1392	80.48	120.72	160.96
.3	40.3062	80.61	120.92	161.22
.4	40.3733	80.75	121.12	161.49
.5	40.4403	80.88	121.32	161.76
.6	40.5075	81.01	121.52	162.03
.7	40.5747	81.15	121.73	162.30
.8	40.6420	81.28	121.93	162.57
.9	40.7093	81.42	122.13	162.84
121.0	40.7766	81.55	122.33	163.10
.1	40.8441	81.69	122.53	163.38
.2	40.9116	81.82	122.74	163.64
.3	40.9791	81.96	122.94	163.92
.4	41.0467	82.09	123.14	164.19
.5	41.1143	82.23	123.34	164.46
.6	41.1820	82.36	123.55	164.73
.7	41.2498	82.50	123.75	165.00
.8	41.3176	82.63	123.95	165.27
.9	41.3855	82.77	124.16	165.54
122.0	41.4534	82.91	124.36	165.81
.1	41.5214	83.04	124.56	166.08
.2	41.5895	83.18	124.77	166.36
.3	41.6575	83.31	124.97	166.63
.4	41.7257	83.45	125.18	166.90
.5	41.7939	83.59	125.38	167.18
.6	41.8622	83.72	125.59	167.45
.7	41.9305	83.86	125.79	167.72
.8	41.9989	84.00	126.00	168.00
.9	42.0673	84.13	126.20	168.27

DEPTH.				
5	6	7	8	9
200.53	240.63	280.74	320.84	360.95
200.86	241.03	281.20	321.38	361.55
201.20	241.43	281.67	321.91	362.15
201.53	241.84	282.14	322.45	362.75
201.87	242.24	282.61	322.98	363.36
202.20	242.64	283.08	323.52	363.96
202.54	243.04	283.55	324.06	364.56
202.88	243.45	284.02	324.60	365.18
203.21	243.85	284.49	325.14	365.78
203.55	244.25	284.96	325.67	366.38
203.88	244.66	285.43	326.21	366.98
204.22	245.06	285.91	326.75	367.60
204.56	245.47	286.38	327.30	368.21
204.90	245.87	286.85	327.83	368.81
205.24	246.28	287.33	328.38	369.42
205.57	246.68	287.80	328.91	370.03
205.91	247.09	288.27	329.36	370.64
206.25	247.50	288.75	330.00	371.25
206.59	247.90	289.22	330.54	371.85
206.93	248.31	289.70	331.08	372.47
207.27	248.72	290.17	331.62	373.08
207.61	249.13	290.65	332.17	373.69
207.95	249.53	291.12	332.71	374.30
208.29	249.94	291.60	333.26	374.91
208.63	250.36	292.08	333.81	375.53
208.97	250.76	292.56	334.45	376.15
209.31	251.17	293.03	334.90	376.76
209.65	251.58	293.51	335.44	377.37
210.00	251.99	293.99	335.99	377.99
210.34	252.40	294.47	336.54	378.60

Diam. in Inches.	DEPTH.			
	1	2	3	4
123.0	42.1358	84.27	126.41	168.55
.1	42.2043	84.41	126.61	168.82
.2	42.2729	84.55	126.82	169.09
.3	42.3416	84.68	127.02	169.36
.4	42.1103	84.82	127.23	169.64
.5	42.4790	84.96	127.44	169.92
.6	42.5479	85.10	127.64	170.19
.7	42.6167	85.23	127.85	170.47
.8	42.6857	85.37	128.06	170.74
.9	42.7547	85.51	128.27	171.02
124.0	42.8237	85.65	128.47	171.29
.1	42.8928	85.79	128.68	171.57
.2	42.9619	85.92	128.89	171.85
.3	43.0312	86.06	129.09	172.12
.4	43.1004	86.20	129.30	172.40
.5	43.1697	86.34	129.51	172.68
.6	43.2391	86.48	129.72	172.96
.7	43.3086	86.62	129.92	173.23
.8	43.3780	86.76	130.13	173.51
.9	43.4476	86.89	130.34	173.79
125.0	43.5172	87.03	130.55	174.07
.1	43.5868	87.17	130.76	174.35
.2	43.6566	87.31	130.97	174.62
.3	43.7263	87.45	131.18	174.90
.4	43.7061	87.59	131.39	175.18
.5	43.8660	87.73	131.60	175.46
.6	43.9360	87.87	131.81	175.74
.7	44.0059	88.01	132.02	176.02
.8	44.0760	88.15	132.23	176.30
.9	44.1461	88.29	132.44	176.58

DEPTH.				
5	6	7	8	9
210.68	252.82	294.95	337.09	379.22
211.02	253.22	295.43	337.63	379.84
211.37	253.64	295.91	338.18	380.46
211.71	254.05	296.39	338.73	381.07
212.05	254.46	296.87	339.28	381.69
212.40	254.87	297.35	339.83	382.31
212.74	255.29	297.84	340.38	382.93
213.09	255.70	298.32	340.94	383.55
213.43	256.12	298.80	341.49	384.17
213.78	256.53	298.29	342.04	384.80
214.12	256.94	299.76	342.58	385.41
214.47	257.36	300.25	343.14	386.04
214.81	257.77	300.73	343.70	386.66
215.16	258.19	301.22	344.25	387.28
215.50	258.60	301.70	344.80	387.90
215.85	259.02	302.19	345.36	388.53
216.20	259.43	302.97	345.91	389.15
216.54	259.85	303.16	346.46	389.77
216.89	260.27	303.65	347.02	390.40
217.24	260.68	304.13	347.58	391.02
217.59	261.10	304.62	348.14	391.65
217.94	261.52	305.11	348.70	392.28
218.28	261.94	305.59	349.25	392.90
218.63	262.36	306.08	349.81	393.53
218.98	262.78	306.57	350.37	393.16
219.33	263.20	307.06	350.93	394.79
219.68	263.62	307.55	351.49	395.42
220.03	264.04	308.04	352.05	396.05
220.38	264.46	308.53	352.61	396.68
220.73	264.88	309.02	353.17	397.31

Diam. in Inches.	DEPTH.			
	1	2	3	4
126.0	44.2162	88.43	132.65	176.86
.1	44.2865	88.57	132.86	177.14
.2	44.3567	88.71	133.07	177.43
.3	44.4271	88.85	133.28	177.71
.4	44.4974	88.99	133.49	177.99
.5	44.5679	89.14	133.70	178.27
.6	44.6384	89.28	133.91	178.55
.7	44.7089	89.42	134.13	178.84
.8	44.7795	89.56	134.34	179.12
.9	44.8502	89.70	134.55	179.40
127.0	44.9209	89.84	134.76	179.68
.1	44.9916	89.98	134.98	179.97
.2	45.0625	90.12	135.19	180.25
.3	45.1334	90.27	135.40	180.53
.4	45.2043	90.41	135.61	180.82
.5	45.2753	90.55	135.83	181.10
.6	45.3463	90.69	136.04	181.38
.7	45.4174	90.83	136.25	181.67
.8	45.4886	90.98	136.46	181.95
.9	45.5598	91.12	136.68	182.24
128.0	45.6311	91.26	136.89	182.52
.1	45.7024	91.40	137.11	182.81
.2	45.7738	91.55	137.32	183.10
.3	45.8452	91.69	137.54	183.38
.4	45.9167	91.83	137.75	183.67
.5	45.9883	91.98	137.96	183.75
.6	46.0599	92.12	138.18	184.24
.7	46.1315	92.26	138.39	184.52
.8	46.2032	92.41	138.61	184.81
.9	46.2750	92.55	138.83	185.10

DEPTH.				
5	6	7	8	9
221.08	265.30	309.51	353.73	397.94
221.43	265.72	310.00	354.29	398.57
221.79	266.14	310.50	354.86	399.21
222.14	266.56	310.99	355.42	399.84
222.49	266.98	311.48	355.98	400.47
222.84	267.41	311.98	356.54	401.11
223.19	267.83	312.47	357.10	401.74
223.55	268.25	312.96	357.67	402.38
223.90	268.67	313.45	358.23	403.01
224.25	269.10	313.95	358.80	403.65
224.61	269.53	314.45	359.37	404.29
224.96	269.95	314.94	359.94	404.93
225.31	270.37	315.43	360.50	405.56
225.67	270.80	315.93	361.06	406.20
226.02	271.22	316.43	361.63	406.84
226.38	271.65	316.93	362.20	407.48
226.73	272.08	317.42	362.77	408.11
227.09	272.50	317.92	363.34	408.75
227.44	272.93	318.42	363.90	409.39
227.80	273.36	318.92	364.48	410.04
228.16	273.79	319.42	365.05	410.68
228.51	274.21	319.91	365.62	411.32
228.87	274.64	320.42	366.19	411.97
229.23	275.07	320.92	366.77	412.61
229.58	275.50	321.42	367.34	413.25
229.94	275.93	321.92	367.90	413.89
230.30	276.36	322.42	368.48	414.54
230.66	276.79	322.92	369.05	415.18
231.02	277.22	323.42	369.62	415.83
231.38	277.65	323.93	370.20	416.48

Diam. in Inches.	DEPTH.			
	1	2	3	4
129.0	46.3468	92.69	139.04	185.39
.1	46.4187	92.84	139.26	185.68
.2	46.4907	92.98	139.47	185.96
.3	46.5627	93.13	139.69	186.25
.4	46.6347	93.27	139.91	186.54
.5	46.7068	93.41	140.12	186.83
.6	46.7790	93.56	140.34	187.12
.7	46.8512	93.70	140.55	187.40
.8	46.9235	93.85	140.77	187.69
.9	46.9958	93.99	140.99	187.98
130.0	47.0682	94.14	141.20	188.27
.1	47.1406	94.28	141.42	188.56
.2	47.2131	94.43	141.64	188.85
.3	47.2857	94.57	141.86	189.14
.4	47.3583	94.72	142.07	189.43
.5	47.4309	94.86	142.29	189.72
.6	47.5037	95.01	142.51	190.02
.7	47.5764	95.15	142.73	190.30
.8	47.6493	95.30	142.95	190.60
.9	47.7222	95.44	143.17	190.89
131.0	47.7951	95.59	143.39	191.18
.1	47.8681	95.74	143.60	191.47
.2	47.9412	95.88	143.82	191.76
.3	48.0143	96.03	144.04	192.06
.4	48.0874	96.17	144.26	192.35
.5	48.1606	96.32	144.48	192.64
.6	48.2339	96.47	144.70	192.94
.7	48.3073	96.61	144.92	193.23
.8	48.3806	96.76	145.14	193.52
.9	48.4541	96.91	145.36	193.82

DEPTH.				
5	6	7	8	9
231.74	278.08	324.43	370.78	417.12
232.10	278.51	324.93	371.35	417.77
232.46	278.95	325.44	371.93	418.42
232.82	279.38	325.94	472.50	419.07
233.18	279.81	326.45	373.08	419.72
233.54	280.24	326.95	373.66	420.36
233.90	280.67	327.45	374.23	421.01
234.26	281.11	327.96	374.81	421.66
234.62	281.54	328.46	375.38	422.31
234.98	281.98	328.97	375.97	422.96
235.34	282.41	329.48	376.54	423.61
235.70	282.84	329.98	377.12	424.26
236.07	283.28	330.49	377.70	424.92
236.43	283.72	331.00	378.29	425.57
236.79	284.15	331.51	378.86	426.22
237.16	284.59	332.02	379.45	426.88
237.52	285.02	332.53	380.03	427.54
237.88	285.46	333.03	380.61	428.18
238.25	285.89	333.54	381.19	428.84
238.61	286.33	334.05	381.78	429.50
238.98	286.77	334.57	382.36	430.16
239.34	287.21	335.08	382.94	430.81
239.71	287.65	335.59	383.53	431.47
240.07	288.08	336.10	384.11	432.13
240.44	288.52	336.61	384.70	432.78
240.80	288.96	337.12	385.28	433.44
241.17	289.40	337.64	385.87	434.11
241.54	289.84	338.15	386.46	434.76
241.90	290.28	338.66	387.04	435.42
242.27	290.72	339.18	387.63	436.09

Diam. in Inches.	DEPTH.			
	1	2	3	4
122.0	48.5276	97.05	145.58	194.11
.1	48.6011	97.20	145.80	194.40
.2	48.6747	97.35	146.03	194.70
.3	48.7484	97.50	146.24	194.99
.4	48.8221	97.64	146.47	195.29
.5	48.8959	97.79	146.69	195.58
.6	48.9697	97.94	146.91	195.88
.7	49.0436	98.09	147.13	196.17
.8	49.1176	98.23	147.35	196.47
.9	49.1916	98.38	147.57	196.76
133.0	49.2656	98.53	147.80	197.06
.1	49.3397	98.68	148.02	197.36
.2	49.4139	98.83	148.24	197.66
.3	49.4881	98.98	148.46	197.95
.4	49.5624	99.12	148.69	198.25
.5	49.6367	99.27	148.91	198.55
.6	49.7111	99.42	149.13	198.84
.7	49.7856	99.57	149.36	199.14
.8	49.8601	99.72	149.58	199.44
.9	49.9346	99.87	149.80	199.74
134.0	50.0093	100.02	150.03	200.04
.1	50.0839	100.17	150.25	200.34
.2	50.1586	100.32	150.47	200.63
.3	50.2334	100.47	150.70	200.93
.4	50.3083	100.62	150.92	201.23
.5	50.3832	100.77	151.15	201.53
.6	50.4581	100.92	151.37	201.83
.7	50.5331	101.07	151.60	202.13
.8	50.6082	101.22	151.82	202.43
.9	50.6833	101.37	152.05	202.73

DEPTH.				
5	6	7	8	9
242.64	291.16	339.69	388.22	436.74
243.01	291.61	340.21	388.81	437.41
243.38	292.05	340.73	389.40	438.08
243.74	292.49	341.24	389.98	438.73
244.11	292.93	341.75	390.58	439.40
244.48	293.38	342.27	391.17	440.06
244.85	293.82	342.79	391.76	440.73
245.22	294.26	343.30	392.34	441.39
245.59	294.70	343.82	392.94	442.05
245.96	295.15	344.34	393.53	442.72
246.33	295.59	344.86	394.12	443.39
246.70	296.04	345.38	394.72	444.06
247.07	296.48	345.90	395.31	444.73
247.44	296.93	346.42	395.90	445.39
247.81	297.37	346.93	396.50	446.06
248.19	297.82	347.46	397.10	446.73
248.56	298.27	347.98	397.69	447.40
248.93	298.71	348.50	398.28	448.07
249.30	299.16	349.02	398.88	448.74
249.67	299.60	349.54	399.47	449.41
250.05	300.05	350.06	400.07	450.08
250.42	300.50	350.59	400.67	450.76
250.79	300.95	351.11	401.26	451.42
251.17	301.40	351.63	401.86	452.10
251.54	301.85	352.16	402.46	452.77
251.92	302.30	352.68	403.06	453.45
252.29	302.75	353.21	403.66	454.12
252.67	303.20	353.73	404.26	454.80
253.04	303.65	354.26	404.86	455.47
253.42	304.10	354.78	405.46	456.15

Diam. in Inches.	DEPTH.			
	1	2	3	4
135.0	50.7584	101.52	152.27	203.03
.1	50.8340	101.67	152.50	203.34
.2	50.9090	101.82	152.73	203.64
.3	50.9842	101.97	152.95	203.94
.4	51.0600	102.12	153.18	204.24
.5	51.1351	102.27	153.41	204.54
.6	51.2110	102.42	153.63	204.84
.7	51.2861	102.57	153.86	205.14
.8	51.3620	102.72	154.09	205.45
.9	51.4374	102.87	154.31	205.75
136.0	51.5132	103.03	154.54	206.05
.1	51.5890	103.18	154.77	206.36
.2	51.6650	103.33	155.00	206.66
.3	51.7410	103.48	155.22	206.96
.4	51.8170	103.63	155.45	207.27
.5	51.8930	103.79	155.68	207.57
.6	51.9690	103.94	155.91	207.88
.7	52.0450	104.09	156.14	208.18
.8	52.1210	104.24	156.36	208.48
.9	52.1972	104.39	156.59	208.79
137.0	52.2740	104.55	156.82	209.10
.1	52.3500	104.70	157.05	209.40
.2	52.4262	104.85	157.28	209.70
.3	52.5030	105.01	157.51	210.01
.4	52.5792	105.16	157.74	210.32
.5	52.6560	105.31	157.97	210.62
.6	52.7324	105.46	158.20	210.93
.7	52.8090	105.62	158.43	211.24
.8	52.8860	105.77	158.66	211.54
.9	52.9630	105.93	158.89	211.85

DEPTH.				
5	6	7	8	9
253.79	304.55	355.31	406.06	456.82
254.17	305.00	355.84	406.67	457.51
254.55	305.45	356.36	407.27	458.18
254.92	305.90	356.89	407.87	458.86
255.30	306.36	357.42	408.48	459.54
255.68	306.81	357.94	409.08	460.22
256.06	307.27	358.48	409.69	460.90
256.43	307.72	359.00	410.29	461.57
256.81	308.17	359.53	410.90	462.26
257.19	308.62	360.06	411.50	462.93
257.57	309.08	360.59	412.10	463.62
257.95	309.53	361.12	412.71	464.30
258.33	309.99	361.66	413.32	464.99
258.71	310.45	362.19	413.93	465.67
259.09	310.90	362.72	414.54	466.35
259.47	311.36	363.25	415.14	467.04
259.85	311.81	363.78	415.75	467.72
260.23	312.27	364.32	416.36	468.41
260.61	312.73	364.85	416.97	469.09
260.99	313.18	365.38	417.58	469.77
261.37	313.64	365.92	418.19	470.47
261.75	314.10	366.45	418.80	471.15
262.13	314.56	366.98	419.41	471.83
262.52	315.02	367.52	420.02	472.53
262.90	315.47	368.05	420.63	473.21
263.28	315.94	368.59	421.25	473.90
263.66	316.39	369.12	421.86	474.59
264.05	316.85	369.66	422.47	475.28
264.43	317.32	370.20	423.09	475.97
264.82	317.78	370.74	423.70	476.67

Diam. in Inches.	DEPTH.			
	1	2	3	4
138.0	53.0394	106.08	159.12	212.16
.1	53.1164	106.23	159.35	212.46
.2	53.1933	106.39	159.58	212.77
.3	53.2703	106.54	159.81	213.08
.4	53.3474	106.69	160.04	213.39
.5	53.4245	106.85	160.27	213.70
.6	53.5017	107.00	160.51	214.01
.7	53.5789	107.16	160.74	214.32
.8	53.6562	107.31	160.97	214.62
.9	53.7335	107.47	161.20	214.93
139.0	53.8109	107.62	161.43	215.24
.1	53.8884	107.78	161.66	215.55
.2	53.9659	107.93	161.90	215.86
.3	54.0434	108.09	162.13	216.17
.4	54.1211	108.24	162.36	216.48
.5	54.1987	108.40	162.60	216.80
.6	54.2765	108.55	162.83	217.10
.7	54.3543	108.71	163.06	217.42
.8	54.4321	108.86	163.30	217.73
.9	54.5100	109.02	163.53	218.04
140.0	54.5880	109.18	163.76	218.35
.1	54.6660	109.33	164.00	218.66
.2	54.7440	109.49	164.23	218.98
.3	54.8222	109.64	164.47	219.29
.4	54.9003	109.80	164.70	219.60
.5	54.9786	109.96	164.93	219.91
.6	55.0569	110.11	165.17	220.23
.7	55.1352	110.27	165.41	220.54
.8	55.2136	110.43	165.64	220.85
.9	55.2921	110.58	165.88	221.17

DEPTH.				
5	6	7	8	9
265.20	318.23	371.27	424.31	477.35
265.58	318.70	371.81	424.93	478.04
265.97	319.16	372.35	425.54	478.74
266.35	319.62	372.89	426.16	479.43
266.74	320.08	373.43	426.78	480.12
267.12	320.54	373.97	427.39	480.82
267.51	321.01	374.51	428.02	481.52
267.90	321.48	375.05	428.63	482.21
268.28	321.94	375.59	429.25	482.90
268.67	322.40	376.13	429.86	483.60
269.06	322.87	376.68	430.49	484.30
269.44	323.33	377.22	431.10	484.99
269.83	323.80	377.76	431.73	485.69
270.22	324.26	378.30	432.34	486.39
270.61	324.73	378.85	432.97	487.09
271.00	325.19	379.39	433.59	487.79
271.38	325.66	379.93	434.21	488.48
271.77	326.12	380.48	434.83	489.19
272.16	326.59	381.02	435.46	489.89
272.55	327.06	381.57	436.08	490.59
272.94	327.53	382.12	436.70	491.29
273.33	328.00	382.66	437.33	491.99
273.72	328.46	383.21	437.95	492.70
274.11	328.93	383.75	438.58	493.40
274.50	329.40	384.30	439.20	494.10
274.89	329.87	384.85	439.82	494.80
275.29	330.34	385.40	440.46	495.51
275.68	330.81	385.95	441.08	496.22
276.07	331.28	386.49	441.70	496.92
276.46	331.75	387.04	442.34	497.63

Diam. in Inches.	DEPTH.			
	1	2	3	4
141.0	55.3706	110.74	166.11	221.48
.1	55.4491	110.90	166.35	221.80
.2	55.5278	111.06	166.58	222.11
.3	55.6064	111.21	166.82	222.42
.4	55.6852	111.37	167.06	222.74
.5	55.7640	111.53	167.29	223.06
.6	55.8428	111.69	167.53	223.37
.7	55.9217	111.84	167.77	223.69
.8	56.0007	112.00	168.00	224.00
.9	56.0797	112.16	168.24	224.32
142.0	56.1587	112.32	168.48	224.64
.1	56.2379	112.48	168.71	224.95
.2	56.3170	112.63	168.95	225.27
.3	56.3963	112.79	169.19	225.58
.4	56.4756	112.95	169.43	225.90
.5	56.5549	113.11	169.67	226.22
.6	56.6343	113.27	169.90	226.54
.7	56.7138	113.43	170.14	226.86
.8	56.7933	113.59	170.38	227.17
.9	56.8729	113.75	170.62	227.49
143.0	56.9525	113.90	160.86	227.81
.1	57.0322	114.06	171.10	228.13
.2	57.1119	114.22	171.34	228.45
.3	57.1917	114.38	171.58	228.77
.4	57.2716	114.54	171.81	229.08
.5	57.3515	114.70	172.05	229.40
.6	57.4314	114.86	172.29	229.72
.7	57.5114	115.02	172.53	230.04
.8	57.5915	115.18	172.77	230.36
.9	57.6716	115.34	173.01	230.68

DEPTH.				
5	6	7	8	9
276.85	332.22	387.59	442.96	498.33
277.25	332.69	388.14	443.59	499.04
277.64	333.17	388.70	444.22	499.75
278.03	333.64	389.24	444.85	500.45
278.43	334.11	389.80	445.48	501.17
278.82	334.58	390.35	446.11	501.88
279.22	335.06	390.90	446.74	502.59
279.61	335.53	391.45	447.38	503.30
280.01	336.01	392.01	448.01	504.01
280.40	336.48	392.56	448.64	504.72
280.80	336.95	393.11	449.27	505.43
281.19	337.43	393.67	449.90	506.14
281.59	337.90	394.22	450.54	506.85
281.98	338.38	394.77	451.17	507.56
282.38	338.85	394.33	451.80	508.28
282.78	339.33	395.89	452.44	509.00
283.17	339.80	396.44	453.07	509.71
283.57	340.28	397.00	453.71	510.43
283.97	340.76	397.55	454.34	511.14
284.37	341.24	398.11	454.98	511.86
284.76	341.71	398.66	455.62	512.57
285.16	342.19	399.22	456.26	513.29
285.56	342.67	399.78	456.90	514.01
285.96	343.15	400.34	457.54	514.73
286.36	343.63	400.90	458.17	515.44
286.76	344.11	401.46	458.81	516.16
287.16	344.59	402.02	459.45	516.88
287.56	345.07	402.58	460.09	517.60
287.96	345.55	403.14	460.73	518.32
288.36	346.03	403.70	461.37	519.04

Diam. in Inches.	DEPTH.			
	1	2	3	4
144.0	57.7518	115.50	173.26	231.01
.1	57.8321	115.66	173.50	231.33
.2	57.9124	115.82	173.74	231.65
.3	57.9927	115.99	173.98	231.97
.4	58.0731	116.15	174.22	232.29
.5	58.1536	116.31	174.46	232.61
.6	58.2341	116.47	174.70	232.94
.7	58.3147	116.63	174.95	233.26
.8	58.3953	116.79	175.19	233.58
.9	58.4760	116.95	175.43	233.90
145.0	58.5567	117.11	175.67	234.23
.1	58.6375	117.27	175.91	234.55
.2	58.7184	117.44	176.15	234.87
.3	58.7993	117.60	176.40	235.20
.4	58.8802	117.76	176.64	235.52
.5	58.9613	117.92	176.88	235.84
.6	59.0423	118.08	177.13	236.17
.7	59.1235	118.25	177.37	236.49
.8	59.2047	118.41	177.62	236.82
.9	59.2859	118.57	177.86	237.14
146.0	59.3672	118.73	178.10	237.47
.1	59.4485	118.90	178.34	237.79
.2	59.5299	119.06	178.59	238.12
.3	59.6114	119.22	178.83	238.44
.4	59.6929	119.39	179.08	238.77
.5	59.7745	119.55	179.32	239.10
.6	59.8561	119.71	179.57	239.42
.7	59.9378	119.88	179.81	239.75
.8	60.0196	120.04	180.06	240.08
.9	60.1014	120.20	180.30	240.40

DEPTH.				
5	6	7	8	9
288.76	346.51	404.26	462.02	519.77
289.16	346.99	404.82	462.66	520.49
289.56	347.47	405.38	463.30	521.21
289.97	347.96	405.95	463.94	521.94
290.37	348.44	406.51	464.58	522.66
290.77	348.92	407.07	465.22	523.38
291.17	349.40	407.64	465.87	524.11
291.58	349.89	408.21	466.52	524.84
291.98	350.37	408.77	467.16	525.56
292.38	350.86	409.33	467.81	526.28
292.79	351.34	409.90	468.46	527.01
293.19	351.82	410.46	469.10	527.73
293.59	352.31	411.03	469.74	528.46
294.00	352.79	411.59	470.39	529.19
294.40	353.28	412.16	471.04	529.92
294.81	353.77	412.73	471.69	530.65
295.21	354.25	413.29	472.34	531.38
295.62	354.74	413.86	472.98	532.11
296.03	355.23	414.44	473.64	532.85
296.43	355.72	415.00	474.29	533.57
296.84	356.20	415.57	474.94	534.30
297.24	356.69	416.14	475.58	535.03
297.65	357.18	416.71	476.24	535.77
298.06	357.67	417.28	476.89	536.50
298.47	358.16	417.85	477.54	537.24
298.87	358.64	418.42	478.19	537.97
299.28	359.14	418.99	478.85	538.70
299.69	359.63	419.57	479.50	539.44
300.10	360.11	420.13	480.15	540.17
300.51	360.61	420.71	480.81	540.91

Diam. in Inches.	DEPTH.			
	1	2	3	4
147.0	60.1832	120.37	180.55	240.73
.1	60.2651	120.53	180.80	241.06
.2	60.3471	120.69	181.04	241.39
.3	60.4291	120.86	181.29	241.72
.4	60.5112	121.02	181.53	242.04
.5	60.5933	121.19	181.78	242.37
.6	60.6755	121.35	182.03	242.70
.7	60.7578	121.51	182.27	243.03
.8	60.8401	121.68	182.52	243.36
.9	60.9224	121.84	182.77	243.69
148.0	61.0048	122.01	183.01	244.02
.1	61.0873	122.17	183.26	244.35
.2	61.1698	122.34	183.51	244.68
.3	61.2524	122.50	183.76	245.01
.4	61.3330	122.67	184.01	245.34
.5	61.4177	122.83	184.25	245.67
.6	61.5005	123.00	184.50	246.00
.7	61.5833	123.17	184.75	246.33
.8	61.6661	123.33	185.00	246.66
.9	61.7490	123.50	185.25	247.00
149.0	61.8320	123.66	185.50	247.33
.1	61.9150	123.83	185.75	247.66
.2	61.9981	124.00	185.99	247.99
.3	62.0812	124.16	186.24	248.32
.4	62.1644	124.33	186.49	248.66
.5	62.2477	124.49	186.74	248.99
.6	62.3310	124.66	186.99	249.32
.7	62.4143	124.83	187.24	249.66
.8	62.4978	124.99	187.49	249.99
.9	62.5812	125.16	187.74	250.32

DEPTH.				
5	6	7	8	9
300.92	361.10	421.28	481.46	541.65
301.33	361.59	421.86	482.12	542.39
301.74	362.08	422.43	482.78	543.12
302.15	362.57	423.00	483.43	543.86
302.56	363.07	423.58	484.09	544.60
302.96	363.56	424.15	484.74	545.34
303.38	364.05	424.73	485.40	546.08
303.79	364.54	425.30	486.06	546.81
304.20	365.04	425.88	486.72	547.56
304.61	365.53	426.45	487.38	548.30
305.02	366.02	427.03	488.03	549.04
305.44	366.52	427.61	488.70	549.78
305.85	367.01	428.18	489.35	550.52
306.26	367.51	428.76	490.02	551.26
306.68	368.01	429.35	490.68	552.02
307.09	368.50	429.92	491.34	552.75
307.50	369.00	430.50	492.00	553.50
307.92	369.50	431.08	492.66	554.25
308.33	370.00	431.66	493.33	554.99
308.75	370.49	432.24	493.99	555.74
309.16	370.99	432.82	494.66	556.49
309.58	371.49	433.41	495.32	557.24
309.99	371.99	433.99	495.98	557.98
310.41	372.49	434.57	496.65	558.73
310.82	372.98	435.15	497.31	559.48
311.24	373.48	435.73	497.98	560.22
311.66	373.99	436.32	498.65	560.98
312.07	374.48	436.90	499.31	561.73
312.49	374.98	437.48	499.98	562.47
312.91	375.49	438.07	500.65	563.23

Diam. in Inches.	DEPTH.			
	1	2	3	4
150.0	62.6648	125.33	187.99	250.66
.1	62.7483	125.50	188.24	250.99
.2	62.8320	125.66	188.50	251.33
.3	62.9157	125.83	188.75	251.66
.4	62.9994	126.00	189.00	252.00
.5	63.0832	126.17	189.25	252.33
.6	63.1671	126.33	189.50	252.67
.7	63.2510	126.50	189.75	253.00
.8	63.3350	126.67	190.01	253.34
.9	63.4190	126.84	190.26	253.68
151.0	63.5030	127.01	190.51	254.01
.1	63.5872	127.17	190.76	254.35
.2	63.6714	127.34	191.01	254.68
.3	63.7556	127.51	191.27	255.02
.4	63.8400	127.68	191.52	255.36
.5	63.9243	127.85	191.77	255.70
.6	64.0087	128.02	192.02	256.03
.7	64.0932	128.19	192.28	256.37
.8	64.1777	128.35	192.53	256.71
.9	64.2623	128.52	192.79	257.05
152.0	64.3470	128.69	193.04	257.39
.1	64.4316	128.86	193.29	257.72
.2	64.5164	129.03	193.55	258.06
.3	64.6012	129.20	193.80	258.40
.4	64.6861	129.37	194.06	258.74
.5	64.7710	129.54	194.31	259.08
.6	64.8560	129.71	194.57	259.42
.7	64.9410	129.88	194.82	259.76
.8	65.0261	130.05	195.08	260.10
.9	65.1112	130.22	195.33	260.44

DEPTH.				
5	6	7	8	9
313.32	375.98	438.65	501.31	563.98
313.74	376.49	439.24	501.98	564.73
314.16	376.99	439.82	502.66	565.49
314.58	377.49	440.41	503.32	566.24
315.00	377.89	440.89	503.89	566.99
315.42	378.50	441.58	504.66	567.75
315.84	379.00	442.17	505.34	568.50
316.26	379.51	442.76	506.01	569.26
316.68	380.01	443.35	506.68	570.02
317.10	380.51	443.93	507.35	570.77
317.52	381.02	444.52	508.02	571.53
317.94	381.52	445.11	508.70	572.28
318.36	382.03	445.70	509.37	573.04
318.78	382.53	446.29	510.04	573.80
319.20	383.04	446.88	510.72	574.56
319.62	383.54	447.47	511.39	575.32
320.04	384.05	448.06	512.06	576.07
320.47	384.56	448.65	512.74	576.84
320.89	385.06	449.24	513.42	577.59
321.81	385.57	449.83	514.10	578.36
321.74	386.08	450.43	514.78	579.12
322.16	386.59	451.02	515.45	579.88
322.58	387.10	451.61	516.13	580.64
323.01	387.61	452.21	516.81	581.41
323.43	388.12	452.80	517.49	582.17
323.86	388.63	453.40	518.17	582.94
324.28	389.14	453.99	518.85	583.70
324.71	389.65	454.59	519.53	584.47
325.13	390.16	455.18	520.21	585.23
325.56	390.67	455.78	520.89	586.00

Diam. in Inches.	DEPTH.			
	1	2	3	4
153.0	65.1964	130.39	195.59	260.78
.1	65.2817	130.56	195.84	261.12
.2	65.3670	130.73	196.10	261.47
.3	65.4523	130.90	196.36	261.81
.4	65.5377	131.07	196.61	262.15
.5	65.6232	131.25	196.87	262.49
.6	65.7088	131.42	197.12	262.83
.7	65.7943	131.59	197.38	263.18
.8	65.8800	131.76	197.64	263.52
.9	65.9657	131.93	197.90	263.86
154.0	66.0514	132.10	198.15	264.20
.1	66.1372	132.27	198.41	264.55
.2	66.2231	132.45	198.67	264.89
.3	66.3090	132.62	198.93	265.24
.4	66.3950	132.79	199.19	265.58
.5	66.4810	132.96	199.44	265.92
.6	66.5671	133.13	199.70	266.27
.7	66.6533	133.31	199.96	266.61
.8	66.7395	133.48	200.22	266.96
.9	66.8257	133.65	200.48	267.30
155.0	66.9120	133.82	200.74	267.65
.1	66.9984	134.00	200.99	267.99
.2	67.0848	134.17	201.25	268.34
.3	67.1713	134.34	201.51	268.68
.4	67.2578	134.51	201.77	269.03
.5	67.3444	134.69	202.03	269.38
.6	67.4311	134.86	202.29	269.72
.7	67.5178	135.03	202.55	270.07
.8	67.6045	135.21	202.81	270.42
.9	67.6913	135.38	203.07	270.76

DEPTH.				
5	6	7	8	9
325.98	391.18	456.37	521.57	586.76
326.41	391.69	456.97	522.25	587.53
326.84	392.20	457.57	522.94	588.30
327.26	392.71	458.16	523.62	589.07
327.69	393.22	458.76	524.30	589.83
328.12	393.73	459.36	524.98	590.61
328.54	394.24	459.96	525.66	591.37
328.97	394.76	460.56	526.35	592.15
329.40	395.28	461.16	527.04	592.92
329.83	395.79	461.76	527.72	593.69
330.26	396.31	462.36	528.41	594.46
330.69	396.82	462.96	529.10	595.23
331.12	397.35	463.56	529.78	596.01
331.55	397.85	464.16	530.47	596.78
331.98	398.37	464.77	531.16	597.56
332.41	398.89	465.37	531.85	598.33
332.84	399.40	465.97	532.54	599.10
333.27	399.92	466.57	533.22	599.88
333.70	400.43	467.17	533.91	600.65
334.13	400.95	467.78	534.60	601.43
334.56	401.47	468.38	535.30	602.21
334.99	401.99	468.99	535.98	602.98
335.42	402.50	469 59	536.67	603.76
335.86	403.03	470.20	537.37	604.54
336.29	403.54	470.80	538.06	605.31
336.72	404.06	471.41	538.75	606.10
337.16	404.59	472.02	539.45	606.88
337.59	405.10	472.62	540.14	607.65
338.02	405.62	473.23	540.83	608.44
338.46	406.15	473.84	541.53	609.22

Gallons	Beer.		Ale.		
	B.	F.	B.	F.	G.
36	1	0	1	0	4
45	1	1	1	1	5
54	1	2	1	2	6
63	1	3	1	3	7
72	2	0	2	1	0
81	2	1	2	2	1
90	2	2	2	3	2
99	2	3	3	0	3
108	3	0	3	1	4
117	3	1	3	2	5
126	3	2	3	3	6
135	3	3	4	0	7
144	4	0	4	2	0
153	4	1	4	3	1
162	4	2	5	0	2
171	4	3	5	1	3
180	5	0	5	2	4
189	5	1	5	3	5
198	5	2	6	0	6
207	5	3	6	1	7
216	6	0	6	3	0
225	6	1	7	0	1
234	6	2	7	1	2
243	6	3	7	2	3
252	7	0	7	3	4
261	7	1	8	0	5
270	7	2	8	1	6
279	7	3	8	2	7
288	8	0	9	0	0
297	8	1	9	1	1
306	8	2	9	2	2
315	8	3	9	3	3
324	9	0	10	0	4
333	9	1	10	1	5
342	9	2	10	2	6
351	9	3	10	3	7

Gallons	Beer.		Ale.		
	B.	F.	B.	F.	G.
360	10	0	11	1	0
369	10	1	11	2	1
378	10	2	11	3	2
387	10	3	12	0	3
396	11	0	12	1	4
405	11	1	12	2	5
414	11	2	12	3	6
423	11	3	13	0	7
432	12	0	13	2	0
441	12	1	13	3	1
450	12	2	14	0	2
459	12	3	14	1	3
468	13	0	14	2	4
477	13	1	14	3	5
486	13	2	15	0	6
495	13	3	15	1	7
504	14	0	15	3	0
513	14	1	16	0	1
522	14	2	16	1	2
531	14	3	16	2	3
540	15	0	16	3	4
549	15	1	17	0	5
558	15	2	17	1	6
567	15	3	17	2	7
576	16	0	18	0	0
585	16	1	18	1	1
594	16	2	18	2	2
603	16	3	18	3	3
612	17	0	19	0	4
621	17	1	19	1	5
630	17	2	19	2	6
639	17	3	19	3	7
648	18	0	20	1	0
657	18	1	20	2	1
666	18	2	20	3	2
675	18	3	21	0	3

Gallons.	Beer.		Ale.		
	B.	F.	B.	F.	G.
684	19	0	21	1	4
693	19	1	21	2	5
702	19	2	21	3	6
611	19	3	22	0	7
720	20	0	22	2	0
729	20	1	22	3	1
738	20	2	23	0	2
747	20	3	23	1	3
756	21	0	23	2	4
765	21	1	23	3	5
774	21	2	24	0	6
783	21	3	24	1	7
792	22	0	24	3	0
801	22	1	25	0	1
810	22	2	25	1	2
819	22	3	25	2	3
828	23	0	25	3	4
837	23	1	26	0	5
846	23	2	26	1	6
855	23	3	26	2	7
864	24	0	27	0	0
873	24	1	27	1	1
882	24	2	27	2	2
891	24	3	27	3	3
900	25	0	28	0	4
909	25	1	28	1	5
918	25	2	28	2	6
927	25	3	28	3	7
936	26	0	29	1	0
945	26	1	29	2	1
954	26	2	29	3	2
963	26	3	30	0	3
972	27	0	30	1	4
981	27	1	30	2	5
990	27	2	30	3	6
999	27	3	31	0	7

Gallons.	Beer.		Ale.		
	B.	F.	B.	F.	G.
1008	28	0	31	2	0
1017	28	1	31	3	1
1026	28	2	32	0	3
1035	28	3	32	1	3
1044	29	0	32	2	4
1053	29	1	32	3	5
1062	29	2	33	0	6
1071	29	3	33	1	7
1080	30	0	33	3	0
1089	30	1	34	0	1
1098	30	2	34	1	2
1107	30	3	34	2	3
1116	31	0	34	3	4
1125	31	1	35	0	5
1134	31	2	35	1	6
1143	31	3	35	2	7
1152	32	0	36	0	0
1161	32	1	36	1	1
1170	32	2	36	2	2
1179	32	3	36	3	3
1188	33	0	37	0	4
1197	33	1	37	1	5
1206	33	2	37	2	6
1215	33	3	37	3	7
1224	34	0	38	1	0
1233	34	1	38	2	1
1242	34	2	38	3	2
1251	34	3	39	0	3
1260	35	0	39	1	4
1296	36	0	40	2	0
1332	37	0	41	2	4
1368	38	0	42	3	0
1404	39	0	43	3	4
1440	40	0	45	0	0
2880	80	0	90	0	0
5760	160	0	180	0	0

A TABLE of Allowances for Brewers for ſtrong and ſmall Beer.

	X			VI			
	l.	*s.*	*d.*	23	*l.*	*s.*	*d.*	23
¼	0	0	6	12	0	0	1	7
½	0	1	1	1	0	0	2	14
¾	0	1	7	13	0	0	3	21
1	0	2	2	2	0	0	5	5
2	0	4	4	4	0	0	10	10
3	0	6	6	6	0	1	3	15
4	0	8	8	8	0	1	8	20
5	0	10	10	10	0	2	2	2
6	0	13	0	12	0	2	7	7
7	0	15	2	14	0	3	0	12
8	0	17	4	16	0	3	5	17
9	0	19	6	18	0	3	10	22
10	1	1	8	20	0	4	4	4
11	1	3	10	22	0	4	9	9
12	1	6	1	1	0	5	2	14
13	1	8	3	3	0	5	7	19
14	1	10	5	5	0	6	1	1
15	1	12	7	7	0	6	6	6
16	1	14	9	9	0	6	11	11
17	1	16	11	11	0	7	4	16
18	1	19	1	13	0	7	9	21
19	2	1	3	15	0	8	3	3
20	2	3	5	17	0	8	8	8
21	2	5	7	19	0	9	1	13
22	2	7	9	21	0	9	6	18

	X		VI	
Br.	*l.*	*s*	*l.*	*s.*
23	2	10	0	10
46	5	00	1	00
69	7	10	1	10
92	10	00	2	00
115	12	10	2	10
138	15	00	3	00
161	17	10	3	10
184	20	00	4	00
207	22	10	4	10
230	25	00	5	00
253	27	10	5	10
276	30	00	6	00
299	32	10	6	10
322	35	00	7	00
345	37	10	7	10
368	40	00	8	00
391	42	10	8	10
414	45	00	9	00
437	47	10	9	10
460	50	00	10	00
483	52	10	10	10
506	55	00	11	00
529	57	10	11	10
552	60	00	12	00
575	62	10	12	10
598	65	00	13	00
621	67	10	13	10
644	70	00	14	00
667	72	10	14	10
690	75	00	15	00

	X		VI	
Bar.	*l.*	*s.*	*l.*	*s.*
713	77	10	15	10
736	80	00	16	00
759	82	10	16	10
782	85	00	17	00
805	87	10	17	10
828	90	00	18	00
851	92	10	18	10
874	95	00	19	00
897	97	10	19	10
920	100	00	20	00
943	102	10	20	10
966	105	00	21	00
989	107	10	21	10
1012	110	00	22	00
1035	112	10	22	10
1058	115	00	23	00
1081	117	10	23	10
1104	120	00	24	00
1127	122	10	24	10
1150	125	00	25	00
1173	127	10	25	10
1196	130	00	26	00
1219	132	10	26	10
1242	135	00	27	00
1265	137	10	27	10
1288	140	00	28	00
1311	142	10	28	10
1334	145	00	29	00
1357	147	10	29	10
1380	150	00	30	00

	X		VI	
Bar.	*l.*	*s.*	*l.*	*s.*
1403	152	10	30	10
1426	155	00	31	00
1449	157	10	31	10
1472	160	00	32	00
1495	162	10	32	10
1518	165	00	33	00
1541	167	10	33	10
1564	170	00	34	00
1587	172	10	34	10
1610	175	00	35	00
1633	177	10	35	10
1656	180	00	36	00
1679	182	10	36	10
1702	185	00	37	00
1725	187	10	37	10
1748	190	00	38	00
1771	192	10	38	10
1794	195	00	39	00
1817	197	10	39	10
1840	200	00	40	00
1863	202	10	40	10
1886	205	00	41	00
1909	207	10	41	10
1932	210	00	42	00
1955	212	10	42	10
1978	215	00	43	00
2001	217	10	43	10
2024	220	00	44	00
2047	222	10	44	10
2070	225	00	45	00

	X	VI			X	VI	
Bar.	*l.*	*s.*	*l.*	*s.*	Bar.	*l.*	*s.*	*l.*	*s.*
2093	227	10	45	10	2783	302	10	60	10
2116	230	00	46	00	2806	305	00	61	00
2139	232	10	46	10	2829	307	10	61	10
2162	235	00	47	00	2852	310	00	62	00
2185	237	10	47	10	2875	312	10	62	10
2208	240	00	48	00	2898	315	00	63	00
2231	242	10	48	10	2921	317	10	63	10
2254	245	00	49	00	2944	320	00	64	00
2277	247	10	49	10	2967	322	10	64	10
2300	250	00	50	00	2990	325	00	65	00
2323	252	10	50	10	3013	327	10	65	10
2346	255	00	51	00	3036	330	00	66	00
2369	257	10	51	10	3059	332	10	66	10
2392	260	00	52	00	3082	335	00	67	00
2415	262	10	52	10	3105	337	10	67	10
2438	265	00	53	00	3128	340	00	68	00
2461	267	10	53	10	3151	342	10	68	10
2484	270	00	54	00	3174	345	00	69	00
2507	272	10	54	10	3197	347	10	69	10
2530	275	00	55	00	3220	350	00	70	00
2553	277	10	55	10	3243	352	10	70	10
2576	280	00	56	00	3266	355	00	71	00
2599	282	10	56	10	3289	357	10	71	10
2622	285	00	57	00	3312	360	00	72	00
2645	287	10	57	10	3335	362	10	72	10
2668	290	00	58	00	3358	365	00	73	00
2691	292	10	58	10	3381	367	10	73	10
2714	295	00	59	00	3404	370	00	74	00
2737	297	10	59	10	6808	740	10	148	00
2760	300	00	60	00	10212	1110	00	222	00

A TABLE of Allowances for Brewers for Ale.

Bar.	l.	s.	d.
¼	0	0	6
½	0	1	1
¾	0	1	8
1	0	2	3
2	0	4	6
3	0	6	9
4	0	9	1
5	0	11	4
6	0	13	7
7	0	15	10
8	0	18	2
9	1	0	5
10	1	2	8
11	1	5	0
12	1	7	3
13	1	9	6
14	1	11	9
15	1	14	1
16	1	16	4
17	1	18	7
18	2	0	10
19	2	3	2
20	2	5	5
21	2	7	8

Br.	l.	s.
22	2	10
44	5	00
66	7	10
88	10	00
110	12	10
132	15	00
154	17	10
176	20	00
198	22	10
220	25	00
242	27	10
264	30	00
286	32	10
308	35	00
330	37	10
352	40	00
374	42	10
396	45	00
418	47	10
440	50	00
462	52	10
484	55	00
506	57	10
528	60	00
550	62	10
572	65	00
594	67	10
616	70	00
638	72	10
660	75	00

Bar.	l.	s.
682	77	10
704	80	00
726	82	10
748	85	00
770	87	10
792	90	00
814	92	10
836	95	00
858	97	10
880	100	00
902	102	10
924	105	00
946	107	10
968	110	00
990	112	10
1012	115	00
1034	117	10
1056	120	00
1078	122	10
1100	125	00
1122	127	10
1144	130	00
1166	132	10
1188	135	00
1210	137	10
1232	140	00
1254	142	10
1276	145	00
1298	147	10
1320	150	00

Bar.	*l.*	*s.*	Bar.	*l.*	*s.*	Bar.	*l.*	*s.*
1342	152	10	2002	227	10	2662	302	10
1364	155	00	2024	230	00	2684	305	00
1386	157	10	2046	232	10	2706	307	10
1408	160	00	2068	235	00	2728	310	00
1430	162	10	2090	237	10	2750	312	10
1452	165	00	2112	240	00	2772	315	00
1474	167	10	2134	242	10	2794	317	10
1496	170	00	2156	245	00	2816	320	00
1518	172	10	2178	247	10	2838	322	10
1540	175	00	2200	250	00	2860	325	00
1562	177	10	2222	252	10	2882	327	10
1584	180	00	2244	255	00	2904	330	00
1606	182	10	2266	257	10	2926	332	10
1628	185	00	2288	260	00	2948	335	00
1650	187	10	2310	262	10	2970	337	10
1672	190	00	2332	265	00	2992	340	00
1694	192	10	2354	267	10	3014	342	10
1716	195	00	2376	270	00	3036	345	00
1738	197	10	2398	272	10	3058	347	10
1760	200	00	2420	275	00	3080	350	00
1782	202	10	2442	277	10	3102	352	10
1804	205	00	2464	280	00	3124	355	00
1826	207	10	2486	282	10	3146	357	10
1848	210	00	2508	285	00	3168	360	00
1870	212	10	2530	287	10	3190	362	10
1892	215	00	2552	290	00	3212	365	00
1914	217	10	2574	292	10	3234	367	10
1936	220	00	2596	295	00	3256	370	00
1958	222	10	2618	297	10	3278	372	10
1980	225	00	2640	300	00	3300	375	00

The foregoing Tables (though very ſhort) will ſhew the exact Duty for any Number of Barrels, from 1 to 3300 and upwards, and their uſe will be rendered very plain and eaſie, by the Directions and Examples following.

1. If the Number of Barrels of ſtrong or ſmall Beer given be leſs than 23, ſeek it in the firſt Column of the firſt Table, and againſt it you have the exact Duty in Pounds, Shillings, Pence, and 23th. parts of a Penny. Thus the Duty of 21 Barrels of ſtrong Beer will be found to be 2 *l.* 5 *s.* 7 *d.* and $\frac{2}{3}$ parts of a Penny, or 3 Farthings, the like for ſmall Beer, *&c.*

2. For any Number above 23, ſeek it in ſome of the Columns of Barrels in the Table, and if you cannot find the ſame Number, take the next leſs, and ſet down the ſum of Mony againſt it, then Subtract the Number found from the Number given, the remainder (which will ever be leſs than 23) being found in the firſt Column of the Table gives you a Sum of Mony, which added to the former is the exact Duty required.

1. Example.

What Sum of Mony will the Duty of 1306 *Barrels of ſtrong Beer amount unto?*

See the Work:

		l.	*s.*	*d.*	*pt.*
Number given———	1306				
Next leſs *per* Table———	1288	= 140	00	0	00
Remainder———	18	= 001	19	1	13
The exact Duty is———		141	19	1	13

That is, 141 *l.* 19 *s* 1$\frac{1}{2}$ *d.*

2. Example.

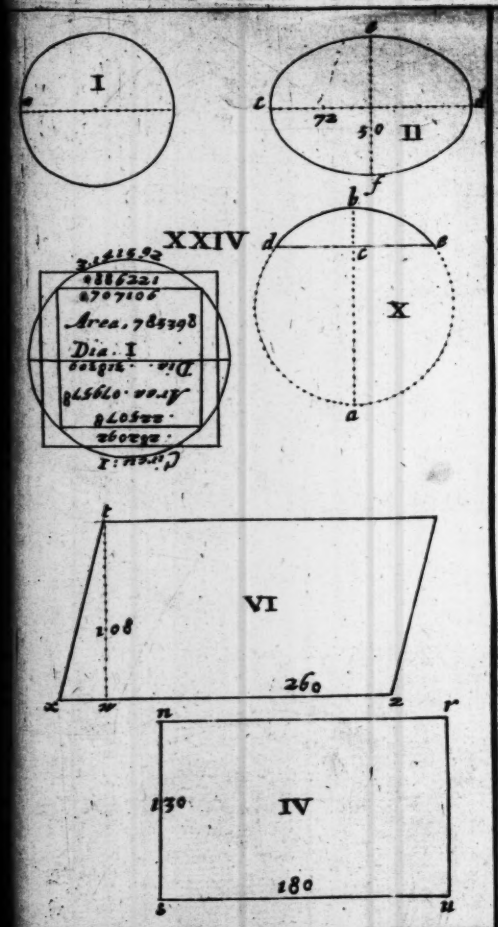
2. Example.

What Sum of Mony will the Duty of 2237 *Barrels of ſtrang Ale amount unto?*

		l.	*s.*	*d.*	*pt.*
The Number given ——	2237				
Next leſs *per* Table ——	2222 =	252	10	0	0
Remainder is —— —— ——	15 =	1	14	1	2
The Duty is —— —— ——		254	04	1	2

Note, The parts in this Table are 22 parts of a Penny, and are reduced into Farthings as the former.

The End.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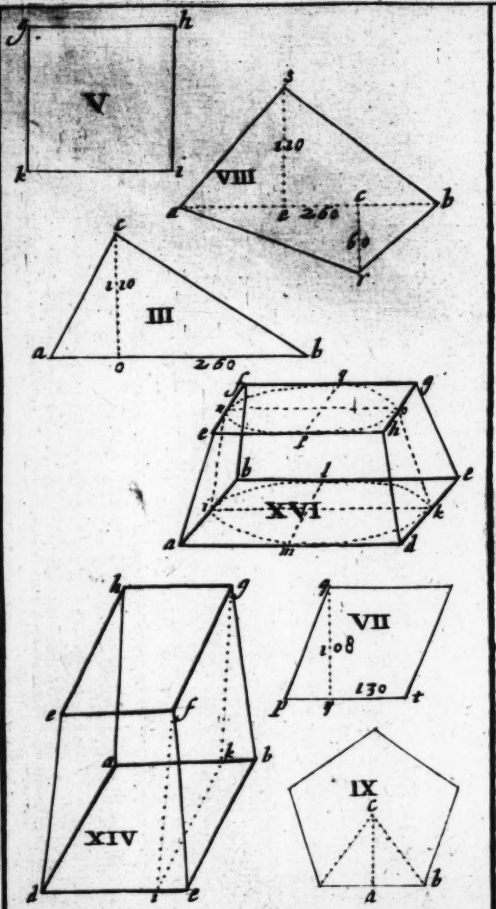
II
72
50
XXIV
Area. 785398
Dia. 1
X
VI
108
260
IV
130
180


V
VIII
120
260
60
III
120
260
XVI
XIV
VII
108
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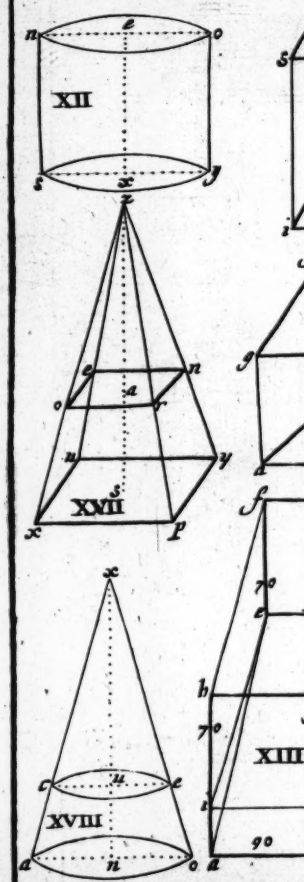
IX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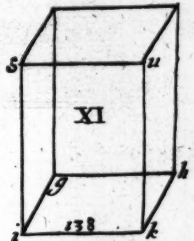

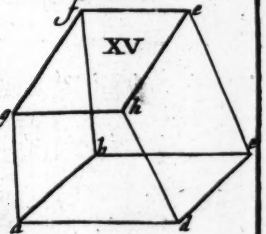

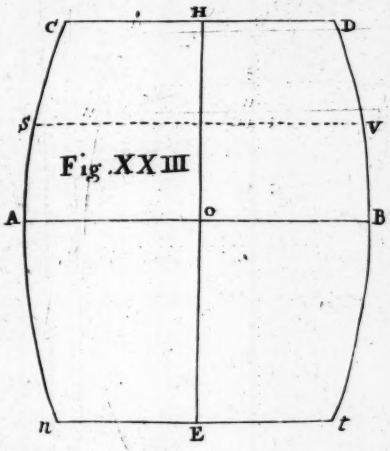

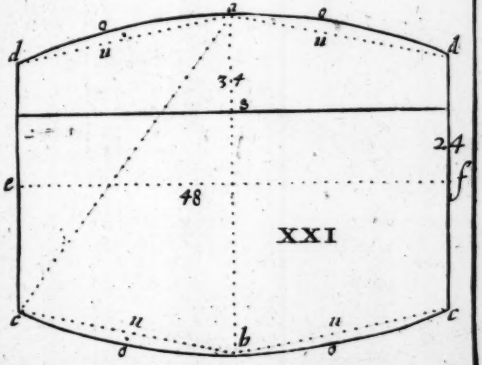

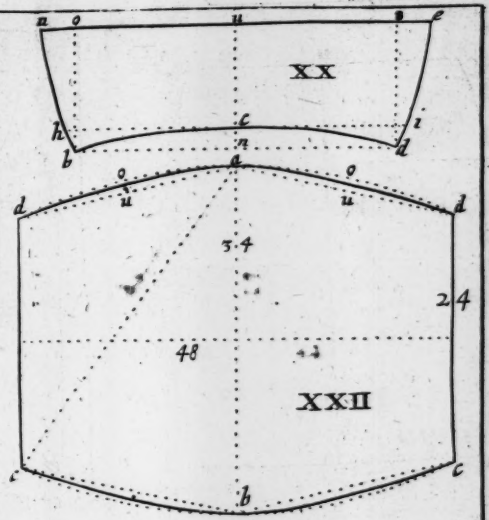

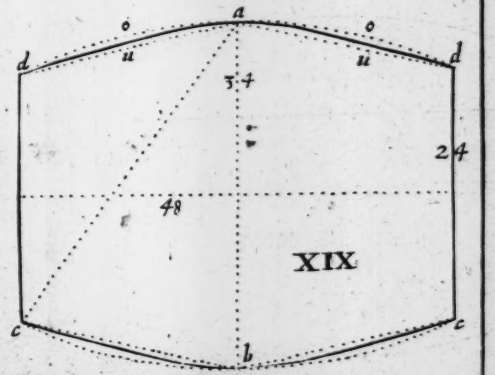

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APPENDIX.

Of Conical Sections.

IN the Doctrine of Conicks there are these two things (very different) to be separately considered. First, What Figures or Curve-Lines do arise from the Section of a Cone by a Plain. And secondly, What is the nature of such Figures or Curve-Lines, considered without respect had to such supposed Original, by cutting a Cone.

See Dr. *Wallis*, *de Con. Sect.* and Treatise of *Algebra*, page 292.

Definition.

1. *A Cone is a Figure made when one of the containing Sides of a Rectangled Triangle remaining fixed, the Triangle is turned round about till it return to the place from whence it first moved: the Axis of a Cone is that fixed Side about which the Triangle is moved: the Base of the Cone is a Circle, which is described by the containing Side that is moved about.*

Of the ſeveral Sections of a Cone.

1. IF a Cone be cut by a Plain through its Axis, the Section will be a Triangle, as S V B. Figure 1.

2. If a Cone be cut by a Plain Parallel to the Baſe, the Section will be a Circle, whoſe Diameter let be S D B. Figure 1. 2. and 3.

3. If a Cone be cut by a Plain that is Parallel to the ſide of the Triangle (S V) the Crooked Section o o A o o is a Parabola; in which P A is the Diameter, or Axis, and D o an Ordinately applied Line. *vide* Figure. 1.

4. If a Cone be cut by a Plain, that will alſo cut one of the ſides of the Triangle produced in E, (Figure 2.) the Crooked Section o o o A o o o is an Hyperbola; in which H A is the intercepted Diameter, A E the Tranſverſe Diameter, and D o an Ordinately applied Line.

5. If a Cone be cut by a Plain through both ſides of the Triangle under the Vertix V, the Crooked Section o o o A o o o (Figure 3.) is an Ellipſis; in which a A is the Diameter, and D o an Ordinately applied Line.

Theſe are all the Sections that can be made by cutting a Cone by a Plain. I ſhall Treat only of the three laſt: and of theſe I ſhall not ſay all that might be ſaid; but ſo much, and in ſuch method as I conceive moſt agreeable to my preſent deſign; which is only to

to explain the Nature and Original of ſome few Solids, which are frequently mentioned in Books (and do ſometime occur in the Practice) of Gaging.

Of the Parabola.

IN the Parabola Figure 1. Let the Cone be cut by a Plain (S D B) Parallel to the Circular Baſe (S P B) it will appear,

1. S D is equal to S P, (for A P is Parallel to S V, and S B is Parallel to S B)

2. S P Multiplied by P B is equal to P o ſquared, *ergo* S D multiplied by D B is equal to D o ſquared.

3. As A P to P o ſquared, So is A D to D o ſquared.

4. *Ergo*, As A P to A D, So is P o ſquared to D o ſquared, which is the general property of a Parabola.

5. Divide the Square of P o by A P, or the Square of D o by A D, in either caſe the Quotient will be the right ſide, for which put L, according to this all the powers of the Ordinates (D o) are proportioned; for whereſoever D be taken, it will be L x A D = D o q, that is, the right ſide multiplied by A D is equal to the Square of D o.

Hence may be found out a way (having the Axis A P, and the right ſide L.)

To deſcribe a Parabola in Plano.

1. THrough the Axis A P (Figure 4.) and at Right Angles to it, let be drawn a competent number (the more the better) of Parallel Lines, as, S S, S S &c. then whereſoever D be taken, it will

be AD x L = Doq, that is, Do is a mean Proportional between AD and the Right Side, this gives the Points o (as many as you please) through which the Crooked Line A o o o being drawn on both sides, you have the Parabola desired.

A second way without knowing the Parameter is this.

To the Lines AP and Po draw the Line Ao, completing the Triangle A P o, then,

AP : Po :: AD : De. and

Po x De = Doq.

A third way.

Draw AN Parallel to Po, divide Po and AN into any Number of equal parts, and divide AP equally to the Square of those parts, as here we divide AN into 6 parts, and AP into 36, then make the first nc = 1, the second nc = 4, the third = 9, the fourth = 16, the fifth = 25 of these 36 parts, so shall the Points o, be all in the Parabola, and through these Points it may be drawn.

Note, The Triangle oAo is $\frac{3}{4}$ of the Parabola ooAoo, and the Parabola ooAoo is $\frac{2}{3}$ of the Parallelogram Noov, these properties agree to any Parabola, as is demonstrated by Dr. *Wallis* in his *Mecanica*, Part. 2. Chap. 5. where 'tis also shewn that if the Semi Parabola PooA, be turned about upon the Line PA, it will describe the Parab. Conoid ooAoo, whose magnitude is equal to half the Circumscribing Cylinder oNVo, that is, a Parabolick Conoid to a Cylinder of the same Base and Altitude is, as 1 to 2, and hence the three following Problems are easily resolved.

Problem

Problem I. Figure 4.

Given oPo *the Diameter of the Baſe, and* AP *the Altitude of a Parabolick Conoid, to find the Solid Content.*

MUltiply the Area of the Baſe by half the Altitude, the Product is the Content ſought, and this will hold whereſoever the Baſe be taken, for if the Parabolick Conoid whoſe Baſe is oPo, and Altitude PA, be cut with a Plain Parallel to the Baſe, (as ſuppoſe oHo) the part cut off, *viz.* oAoH is a true Parabolick Conoid, having all the affections of the other (of which it was a part) and therefore its Content is equal to half the Circumſcribing Cylinder Onro, and hence the Content of the Fruſtum oHooPo may be alſo obtained; for the Parabolick Conoid whoſe Baſe is oPo, leſs the Paraboliek Conoid, whoſe Baſe is oHo, is equal the Fruſtum oHooPo, and thus the Content of the Fruſtum ooLoo, equal the Fruſtum oBGo, may be found, and theſe two Fruſtums ooLoo, and oGBo, abutting upon the common Baſe oPo, are what ſeveral Writers would have to repreſent a Cask, but 'tis obvious, from the Nature and Conſtruction of the Figure, that no Cask can be in this form: I ſhall not therefore examine whether the Rules they give for finding the Content of ſuch (ſuppoſed) Cask be true or falſe; for let that be as it will, they are of little or no uſe in the buſineſs of Cask Gaging.

But if a Tun ſhall be in the form of the Fruſtum of a Parabolick Conoid, the Content of the whole, and alſo upon every Inch of ſuch Tun may be readily found by what hath been already ſaid, and ſhall be further declared in the two next Problems.

Problem II. Figure 4.

Let B o P o G *repreſent a Tun in the form of the Fruſtum of a Parabolick Conoid, whoſe Content is ſought.*

TO the Area of the greater Baſe o P o add the Area of the leſſer Baſe B L G, then Multiply their ſum by half the depth or Altitude (*viz.* $\frac{1}{2}$ P L) the Product is the Content. Or,

Get the Area of the Circle in the middle of the depth (*viz.* v y) this Multiplied by P L gives the Content of the whole Tun.

Problem

Problem III. Figure 4.

There is a Tun in the form of the Fruſtum of a Parabolick Conoid, o H o *the Diameter of the leſſer Baſe is* 20 *Inches,* o P o *the Diameter of the greater Baſe* 40, *and* H P *the depth* 30, *to find the Content of this Tun upon every Inch.*

BY (o P o) the Diameter of the greater Baſe, divide the Square of (o H o) the Diameter of the leſſer Baſe, the Quotient is equal to Z H e, ſubtract this from the Diameter of the greater Baſe, and by the remainder divide the Rectangle of the greater Diameter o P o, and Tuns depth H P, the Quotient is equal to A P the Axis of the Parabolick Conoid, by this Axis divide the Square of the Diameter of the greater Baſe, the Quotient reduced into Ale Gallons (by dividing it by 359.05) is the difference between the Content of the firſt and ſecond Inches of the Tun; by this difference multiply the Axis leſs .5, it gives the Content of the firſt Inch next the greater Baſe; from which ſubtract the ſaid difference, there will remain the Content of the ſecond Inch, and from this ſubtract as above, there reſt the third Inch, and ſo of the reſt till you come to the leſſer Baſe.

Example.

The Square of o H o is 400, this divided by o P o *viz.* 40, quotes 10, equal to Z H e, this taken from o P o 40, there reſt 30, by this 30 divide the Rectangle of o P o 40, into H P 30, (that is 1200)

the Quote is 40, equal A P the Axis of the Parabolick Conoid, the Square of o P o 40, is 1600, this divided by the Axis 40, gives 40, and this divided by 359.05 Quotes .1114 of an Ale Gallon, and such is the difference between the Content of the first and second Inches of this Tun. Lastly, The Axis A P 40 less .5 is 39.5, this multiplied by the difference last found, gives 4.4004 the Content of the first Inch of the Tun next the greater Base, from which the abovesaid difference (*viz.* .1114) being Subtracted, there rest 4.2890 the Content of the second Inch, and so by a continued Subtraction of the said difference, the Content upon every Inch is found as in this following Table, these added together make 83,5530 the whole Content of the Tun, and if we find the whole Content of this Tun by the two Problems aforegoing, it will be the same with this, *viz.* 83,553

Diam.	Area's.	
o P o = 40	4.4562	
o H o = 20	1.1140	
	5.5702	
½ H P =	15	
	278510	
	55702	
Content =	83.5530	Ale Gallons.

A

A Table shewing how many Ale Gallons the foregoing Tun will contain upon every Inch of its depth.

Tuns Depth	Content upon every Inch.	Com. Differ.
1	4.4004	
2	4.2890	
3	4.1776	.1114
4	4.0662	.1114
5	3.9548	.1114
6	3.8434	.1114
7	3.7320	.1114
8	3.6206	.1114
9	3.5092	.1114
10	3.3978	.1114
11	3.2864	.1114
12	3.1750	.1114
13	3.0636	.1114
14	2.9522	.1114
15	2.8408	.1114
16	2.7294	.1114
17	2.6180	.1114
18	2.5066	.1114
19	2.3952	.1114
20	2.2838	.1114
21	2.1724	.1114
22	2.0610	.1114
23	1.9496	.1114
24	1.8382	.1114
25	1.7268	.1114
26	1.6154	.1114
27	1.5040	.1114
28	1.3926	.1114
29	1.2812	
30	1.1698	
	83.5530	

Note, From this Table may be Collected another, that shall shew the Number of Gallons contained in this Tun at any depth, as hath been already done in Sect. VIII. of the foregoing Tract.

Of a Parabolick Spindle.

IF the Parabola G A H (Fig. 5.) ſhall be turned round about upon the right Line G H, it will deſcribe a Solid as G A H B G, which Solid is called a Parabolick Spindle.

Now every Parabolick Spindle is to its circumſcribing Cylinder, as 8 to 15. (*vid.* Dr. *Wallis*, *Mechanica* Part. 2. Chap. 5. Probl. 12.)

Therefore the Parabolick Spindle G A H B G, is $\frac{8}{15}$ of the Cylinder I K M L.

Problem IV. Figure 4.

To Calculate the Diameters, and thereby to find the Content of the middle Fruſtum of a Parabolick Spindle, whoſe greateſt Diameter A B *is* 32 *Inches, the leſſer* C D 24. *and the length* P S 40. (*or of the whole Spindle* G A H B D.)

1. DIvide the three given Diameters in the middle in the Points P Q S: through theſe Points draw the ſtraight Line Z Z, make b E Parallel to Q S, then ſhall A b be 4. A Q 16, and b e 20; and becauſe it is a Parabola it will be

A b : A Q : : b E q : Q H q that is,
4 : 16 : : 400 : 1600

the Square Root of 1600 is 40 equal to Q H.

2. Now to find a Diameter in the middle of every Inch of G Q (or P Q) through the Point A draw I K parallel to G H and equal to it; let I A and

and G H, be each divided into half Inches, which here will be 80, the Square of this is 6400, by which Divide A Q 16, the Quotient will be .0025, this Multiplied by the Squares of all the odd Numbers, *viz.* 1. 3. 5. 7, *&c.* to 79, the ſeveral Products give the length of the lines n e, which taken from the Line A Q leaves the Lines e o the Semidiameters of the Parabolick Spindle, one in the middle of every Inch of G Q. But to contract this Work, we need but the three firſt Diameters, the reſt are obtained by Subtracting the Common difference, as will appear by the following Operation.

.0025	1	is	.0025	each of
Multiplied by	9		.0225	
	25		.0625	

theſe taken from A Q 16 the three remainders will

be	15.9975	theſe Multiplied by 2 gives——	31.995
	15.9775		31.955
	15.9375		31.875

the three Diameters, one in the middle of every Inch next to A B, to find the Common Difference, take the ſecond Diameter from the firſt there reſt .040; take the third from the ſecond, the remainder is .080, take .040 from .080 there reſt .040, the ſecond difference, which is the ſame throughout, and by this Common Difference the reſt of the Diameters are thus found. Take the firſt difference (*viz.* .040) from the firſt Diameter, the remainder is the ſecond Diameter; add the ſecond Difference to the firſt difference, and Subtract their ſum from the ſecond Diameter, the remainder will be the third Diameter, and ſo of the reſt, as in the following Table;

ble; in the first Column of which you have the true Diameters, one in the middle of every Inch of G Q. In the second Column are the first Differences, in the third, the second Difference, which is the same throughout, in the last Column you have the Area's of the several Circles in Ale Gallons, or (which is the same thing) the Content upon every Inch of the half Spindle A G D B A, the sum of these is 60.817, this doubled is 121.634, and such is the Content of this Parabolick Spindle: and for confirmation of this Rule, it may be observed, that the Content here found (*viz.* 121.634) is 8 fifteen parts of the Cylinder I K M L, whose Diameter A B is 32, and length G H 80 Inches, for the Area of 32 is 2.851, this Multiplied by 80, is 228.08, the Content of the Cylinder, this Divided by 15, gives 15.205, and this Multiplied by 8, is 121.64, which agrees with what we have cited above out of Dr. *Wallis*.

Diameters,

	Diameters.	First Diff.	Sec. Diff.	Area's or Con.
1	31.995	40		2.851
2	31.955	80	40	2.844
3	31.875	120	40	2.829
4	31.755	160	40	2.808
5	31.595	200	40	2.780
6	31.395	240	40	2.747
7	31.155	280	40	2.704
8	30.875	320	40	2.656
9	30.555	360	40	2.610
10	30.195	400	40	2.540
11	29.795	440	40	2.472
12	29.355	480	40	2.400
13	28.875	520	40	2.322
14	28.355	560	40	2.224
15	27.795	600	40	2.150
16	27.195	640	40	2.060
17	26.555	680	40	1.963
18	25.875	720	40	1.862
19	25.155	760	40	1.761
20	24.395	800	40	1.658
21	23.595	840	40	1.549
22	22.755	880	40	1.442
23	21.875	920	40	1.332
24	20.955	960	40	1.222
25	19.995	1000	40	1.112
26	18.995	1040	40	1.002
27	17.955	1080	40	0.895
28	16.875	1120	40	0.791
29	15.755	1160	40	0.691
30	14.595	1200	40	0.592
31	13.395	1240	40	0.500
32	12.155	1280	40	0.412
33	10.875	1320	40	0.328
34	9.555	1360	40	0.255
35	8.195	1400	40	0.187
36	6.795	1440	40	0.128
37	5.355	1480	40	0.080
38	3.875	1520	40	0.041
39	2.355	1560	40	0.015
40	0.795			0.002
			The Sum is	60.817

The ſum of the firſt 20 Inches is 48.241, and ſo much is the Content of the Fruſtum CABD, this doubled gives 96.49, the Content of CAEFBD, the middle Fruſtum of the Parabolick Spindle, which was required.

Note, The middle Fruſtum CAEFBD, may repreſent a Cask.

And if a Cask be in this Form, and the Diameters and length as above ſuppoſed, its Content is 96.49 Ale Gallons.

Whereas by Mr. *Jordaine's* Rule in his *Duodecimal Arithmetick*, page 291. it would be but 94.108.

And by Mr. *Hunt's* Rule, in his *Gagers Magazine*, page 257, it will be but 93.877.

And (which is ſtill further from the truth) it would be but 87.936, by Mr. *Hunt's* Rule, in his *Practical Gaging Epitomized* page 72. which is more than 8 Gallons too little.

Of the Hyperbola.

IN Fig. 2. the Cone SVB is cut by a Plain HA, which alſo cuts one ſide of the Triangle produced in E, therefore the Crooked Section oooAooo is an Hyperbola, EA is the Tranſverſe Diameter, and AH the intercepted Diameter. Now if the Cone be cut by another plain Parallel to the Baſe, this Section will be a Circle, whoſe Diameter is SDB: and SD x DB = Doq, therefore Do is an Ordinately applied Line, and if Do be a Line Ordinately applied, it will be

AH x EH : AD x ED :: Hoq : Doq

In

In words thus,

As A H Multiplied by E H, is to A D Multiplied by E D.

So is H o Squared, to D o Squared.

And if we make it

As A D Multiplied by E D, is to D o Squared.

So A E to L, the fourth term L will be the Right Side, with the help of which the Ordinates may be all found by this Proportion.

As the Tranſverſe Diameter is to the Right Side.

So is the Rectangle under the Intercepted Diameter, and the Sum of the Tranſverſe Diameter and Intercepted Diameter, to the Square of the Ordinate ſought.

Therefore if the Tranſverſe Diameter A E, Intercepted Diameter A D (in Fig. VI.) and the Right Side be given the Hyperbola may eaſily be deſcribed in Plano, for the Ordinates D o may be all found the proportion above, which gives the points o o o, through which the Hyperbola muſt be drawn.

Of an Hyperbolick Conoid.

IF the Semi-Hyperbola A S S S D be turned round about upon the Line A D, it will deſcribe the Hyperbolick Conoid S S A o o, and the Content of this Hyperbolick Conoid may be eaſily found; for if we put the Tranverſe Diameter = t, the Intercepted Diameter = d, the magnitude of the Hyperbolick Conoid = A, and the Magnitude of a Cylinder of the ſame Baſe and Altitude = B, it will be

$3t + 2d : 6t + 6d :: A : B.$ therefore
$6t + 6d : 3t + 2d :: B : A.$

In

In words thus,

1. To 6 times the Tranſverſe Diameter A E, add 6 times the Intercepted Diameter A D, the Sum is your Diviſor.

2. To 3 times the Tranſverſe Diameter, add twice the Intercepted Diameter, by this Sum Multiply the Content of the Cylinder N V O S, the Product is your Dividend; which Divided by the Diviſor above found, the Quotient will be the Content of the Hyperbolick Conoid, whoſe Baſe S D o, is the Baſe of the Cylinder N V O S, and Altitude A D equal to S N the Altitude of the Cylinder, and this will hold whereſoever the Baſe be taken; for ſuppoſe the firſt S D o next to the vertex A, be taken for the Baſe of the Hyperbolick Conoid S A o, (and of the Cylinder S n u o,) the Content of this Hyperbolick Conoid may be found by this Rule, and ſo the Content of any Fruſtum, S D o o D S may be likewiſe found, for the little Hyperbolick Conoid S A o, taken from the Hyperbolick Conoid S S S A o o o, there remains the Fruſtum S D o o D S, and thus the Content of the whole, and alſo the Content of any Fruſtum, and conſequently of every Inch of the Hyperbolick Conoid may be readily found, as will appear in the following Example.

Example Fig. VI.

Let A E the Tranſverſe Diameter, and A D the Intercepted Diameter be each 32 Inches, and S D o the Diameter of the Baſe 45.255 Inches, to find the Content of the Hyperbolick Conoid in Ale Gallons.

1. Six times A E is 192, and 6 times A D is 192, e Sum of theſe is 384, for the firſt term of the throportion.

2. Three

2. Three times AE is 96, and twice AD is 64, the Sum of theſe is 160 for the ſecond term.

3. The Diameter of the Baſe SDo is 45.255 (or the Square Root of 2048) the Area of this Circle is 5.7039 Ale Gallons, this Multiplied by the Altitude AD or NS, *viz.* 32 gives 182.524 the Content of the Cylinder NSoV, and this is the third term of the Proportion, which will ſtand thus,

384 : 160 :: 182.524 :

Multiply the third by the ſecond, and Divide the Product by the firſt, the Quotient or fourth term will be 76.05, and ſuch is the Content of the Hyperbolick Conoid in Ale Gallons.

To find the Content of any Fruſtum SDo, oDS, *of the Hyperbolick Conoid, Fig. VI.*

BY the directions above the Content of the little Hyperbolick Conoid, whoſe Altitude ſn is 4, and ſo the Diameter of its Baſe 12 Inches, will be found to be .7724 parts of a Gallon, this taken from the Content of the Hyperbolick Conoid above found, *viz.* 76.05, the remainder is 75.2776, the Content of the Fruſtum SDo, oDS.

And thus the Content of every Inch may be found: for let the firſt AD be 1 Inch, the ſecond 2 Inches, and the third 3 &c. by the Rule above given

the 1 2 3	Hyperb. Conoid whoſe Altitude is	1 Inch 2 Inches 3 Inches	will be found to be	.04549 .18567 .42611

The firſt of theſe, *viz.* .04549 taken from the ſecond,

ſecond, the remainder is .14018 the Content of the ſecond Inch S D o, o D r, in like manner the ſecond Hyperbolick Conoid, *viz.* .18567 taken from the third, there will remain .24044 the Content of the third Inch r D o, o D S.

Having thus got the Content of the three firſt Inches, a Table may be made by Addition for the whole Hyperbolick Conoid thus.

Set down the three firſt Inches as in the ſecond Column of this Table, then take the firſt Inch from the ſecond, the difference is .09469, ſet this in the third Column, take the ſecond Inch from the third, the difference is .10026, ſet this in the third Column under the other, then take .09469 from .10026 the difference is .00557, and this will be the ſame throughout, as in the fourth Column.

Now this .00557 added to .10026 is .10583 (the third Number in the third Column) and this added to the third Inch gives .34627 the Content of the fourth Inch, and ſo by a continual Addition the Table is made.

Now ſuppoſe the Hyperbolick Conoid, whoſe Altitude is 32 Inches, were divided into ſo many parts of equal thickneſs, by Plains S D o parallel to the Baſe, the Content of every of theſe parts are expreſs'd by the Numbers in the ſecond Column of this Table, and the Sum of all theſe is 76.04912, which is the true Content of the Hyperbolick Conoid within leſs than $\frac{1}{1000}$ of a Gallon.

Inches

Inches.	Content of every Inch of the Hyp. Conoid.	First Differ.	Second Differ.
1	.04549	.09469	
2	.14018	.10026	.00557
3	.24044	.10583	.00557
4	.34627	.11140	.00557
5	.45767	.11697	.00557
6	.57464	.12254	.00557
7	.69718	.12811	.00557
8	.82529	.13368	.00557
9	.95897	.13925	.00557
10	1.09822	.14482	.00557
11	1.24304	.15039	.00557
12	1.39343	.15596	.00557
13	1.54939	.16153	.00557
14	1.71092	.16710	.00557
15	1.87802	.17267	.00557
16	2.05069	.17824	.00557
17	2.22893	.18381	.00557
18	2.41274	.18938	.00557
19	2.60212	.19495	.00557
20	2.79707	.20052	.00557
21	2.99759	.20607	.00557
22	3.20368	.21166	.00557
23	3.41534	.21723	.00557
24	3.63257	.22280	.00557
25	3.85537	.22837	.00557
26	4.08374	.23394	.00557
27	4.31768	.23951	.00557
28	4.55719	.24508	.00557
29	4.80227	.25065	.00557
30	5.05292	.25622	.00557
31	5.30914	.26179	.00557
32	5.57093		

76.04912 = Con. of the Hyp. Con.

To

To Describe an Hyperbola in Plano Geometrically.

FRom the middle of the Right Line R R (Fig. VII.) and at Right Angles to it; draw the Line W A B, then divide W R into any Number of equal parts, and draw the Lines r o b parallel to W B, this done set one point of the Compasses in A, and extend the other to r, where keep it fixt whilst the other point is applied to the Line r b, it will give the point o, and thus if all the Lines A r be transferred to the Lines r b, from r towards b, they will give the points o o o through which the Crooked Line O o o A o O, being drawn you have the Hyperbola required, in which W A is half the Transverse Diameter, and A B the Intercepted Diameter (*vid.* Dr. *Wallis's Mecanica*, Part 2. pag. 553.)

Of the Hyperbolick Spindle.

IF the Hyperbola O o o A o O (Fig. VII.) be turned round about upon the Right Line O B O it will describe the Solid O o A o O E D E, which Solid is called a Hyperbolick Spindle.

Now supposing the Plain O A D, to be made up of innumerable Right Lines parallel to A D, whereof one is o e, the quantity of these Lines may be found as shall be shewn below, and although in Mathematical strictness, Lines cannot fill a Plain by reason they have no breadth, yet the quantity of the Lines o e added together (supposing those Lines innumerable) is so near the true Area of the Plain O A D that the difference will be less than any assignable quantity.

In

In like manner ſuppoſing the Semi Hyperbolick Spindle O A D to be made up of innumerable Circles, whoſe Diameters are the Lines o e, the Content of all theſe Circles added together, is near equal to the Content of the Semi-Hyperbolick Spindle O A D, and if theſe Circles be infinitely many, the difference will be infinitely ſmall, or leſs than any aſſignable. (*vid.* Dr. *Wallis's Treatiſe of Algebra* page 286.) The length of the Lines o b may be thus found.

To the Square of W A add the Square of W r, the Sum is the Square of r o, (per 47. 1 Eucl.) and the Square-Root of r o, that is, the Line r o it ſelf, taken from the Line W B, the remainder is the Line o b, this doubled is the Diameter or Line o e, and this will hold whereſoever the point r be taken, and ſo innumerable Diameters o e may be found on both ſides A D towards o, and conſequently the Circles filling the Hyperbolick Spindle, and therefore the Spindle it ſelf being made up of theſe its Content is alſo known.

But for our preſent purpoſe it will be ſufficient to find a Diameter in the middle of every Inch of O B, and then we ſhall have ſo many Circular Plains of one Inch thick (which here we take as Cylinders) the Content of theſe added together, is the Content of the Semi-Hyperbolick Spindle as near as is requiſite in ordinary Practice, I ſhall indeavour to illuſtrate what hath been ſaid, by ſhewing the Content of the middle Fruſtum E O A o E D in Ale Gallons.

Example.

Let W A be 48 Inches, A B 16, W B 64, the length of the Fruſtum b b = E E 40, the greateſt Diameter A D 32, and the leſſer E o 24 Inches.

Suppoſe

Suppoſe the firſt W r = .5 the Square of W A 48 is 2304, to this add the Square of W r, *viz.* .25 the Sum is 2304.25.

Let the ſecond W r be 1.5 the Square of this *viz.* 2.25 added to 2304 is 2306.25, and ſo of the reſt as in the following Table,

In the firſt Column of which you have the Squares of W r, W r; r being taken in the middle of every Inch of the firſt 20 Inches from W towards R.

In the ſecond Column are the Sum of the Squares of W A and W r, that is, the Squares of A r or r o.

In the third Column is the Square Roots of the Numbers in the ſecond, that is, the length of the Lines r o, each of theſe Numbers taken from W B 64, leaves the Lines o b, which doubled are the Diameters o e, as in the fourth Column.

In the laſt Column are the Areas of theſe Diameters in Ale Gallons, or the Content upon every Inch of the Portion or Fruſtum A D E o, the Sum of theſe is 48.062, which double is 96.124, the Content of the middle Fruſtum o E D E o A which was required.

Whereas by Mr. *Hunt's* Rule in his *Gagers Magazine* page 257, the Content of this Fruſtum would be but 90.31 Ale Gallons, which is about 6 Gallons too little.

A Table shewing the Content of every Inch of the middle Frustum of the Hyperbolick Spindle E o A o E D (Fig. VII.) in Ale Gallons.

Squares of W r.	Sum of the Squares of W A and W r.	Square-Roots of the Squar of A r or r o, or the length of the lines r o.	Diameters o e.	Area's of the Circles o e.
.25	2304.25	48.003	31.994	2.850
2.25	2306.25	48.023	31.954	2.836
6.25	2310.25	48.065	31.870	2.820
12.25	2316.25	48.127	31.746	2.800
20.25	2324.25	48.210	31.580	2.772
30.25	2334.25	48.314	31.372	2.738
42.25	2346.25	48.438	31.124	2.698
56.25	2360.25	48.582	30.836	2.649
72.25	2376.25	48.747	30.506	2.591
90.25	2394.25	48.931	30.138	2.528
110.25	2414.25	49.135	29.730	2.460
132.25	2436.25	49.358	29.284	2.386
156.25	2460.25	49.601	28.798	2.308
182.25	2486.25	49.862	28.276	2.225
210.25	2514.25	50.142	27.716	2.138
240.25	2544.25	50.440	27.120	2.047
272.25	2576.25	50.757	26.486	1.952
306.25	2610.25	51.091	25.818	1.854
342.25	2646.25	51.442	25.116	1.755
380.25	2684.25	51.810	24.380	1.655

Sum is ——— 48.062
This doubled is ——— 96.124
The Content of Frustum E o A o E D.

Of an Ellipſis.

IF the Cone S e B d V (Fig. 3.) be cut by a Plain through both the ſides of the Triangle under the Vertex V, the Crooked Section a o o A o o is an Ellipſis, in which A a is the Tranſverſe Diameter or Axes, and A D a Diameter intercepted Diameter.

Let the Cone be cut by another Plain (S B) parallel to the Baſe S e B d, it will be S D x D B = D o q (per 35. 3. *Eucl.*) that is, S D Multiplied by D B, is equal to the Square of D o, and this will hold whereſoever D be taken, and give the quantity of the Lines D o, which are called Ordinately applied Lines, and the Lines o D o, which are ſuppoſed to cut the Line A a, at Right-Angles are called the Conjugate Diameters of the Ellipſis.

To Deſcribe an Ellipſis in Plano, the Tranſverſe Diameter A E, *and the Conjugate Diameter* H T, *or the Ordinate* S T, *being given.*

Figure VIII.

DRaw ſtraight Lines parallel to H T, as h b t, O B P, &c. then,

As A S Multiplied by E S, is to A B Multiplied by E B,

So is the Square of S T to the Square of B P.

This will hold whereſoever B P be taken, and ſo a competent number of Points, P being found a Curved Line, A P P E drawn through thoſe Points, and

and continued the same on the other side (as H g H h A) you have the Ellipsis required.

Or thus,

Let it be,
As A S Multiplied by E S is to the Square of S T,
So is A E to E L, (E L is called the *Latus Rectum* or Right Side) draw the Line E L Parallel to S T, and joyn A L, then
A B Multiplied by B r, will be equal to the Square of B p.

Or thus,

As A E to E L:
So is A B Multiplied by B E to the Square of B P.

Another way may be this,

With the Radius A S describe the Quadrant A d D, and let the Lines T H, t h, be continued till they touch the Circle in D and d,
It will be S d q — S b q = b d q (per 47. 1. Eucl. then,
S D q : S H q :: b d q : b h q, this gives the point h, through which the Ellipsis must be drawn.

Or Mechanically thus,

Take in the Compasses, the extent S A or S E, set one Foot in H, and bring the other to the Line A E, it will give the points n n, these points are called the *Nodes* of the Ellipsis.

Now it is evident from the conſtruction of the Figure, that the diſtance from the Center S to the Circumference of the Circle (d or D) and from thence to the Center again, is equal to the Diameter of the Circle; and alſo the diſtance from one Node to the Ellipſis (in H) and from thence to the other Node, is equal to twice the Radius S D, or to the Line A E: therefore,

As by tying a thread (twice the length of the Radius) about a fixed point in the Center, drawing another point in the extremity about, it will deſcribe a Circle: So by tying a thread about three Pins fixed in the points n n and H, and carrying the point H about in the extremity of the thread (the other two points remaining fixed) it will deſcribe the Ellipſis, for as S D S is equal to S d S: So n H n is equal to n g n.

Of a Spheroid.

IF the Semi-Ellipſis A t T P E be turned round about the Line A E, it will deſcribe the ſolid Figure A t P E g H h, which Solid is called a Spheroid.

1. The Content of every Spheroid is equal $\frac{2}{3}$ of the Content of a Cylinder, whoſe Baſe is equal to the greateſt Circle of the Spheroid, and its Altitude the ſame; therefore the Area of the greateſt Circle of a Spheroid Multiplied by $\frac{2}{3}$ of the length gives the Content.

2. If the Spheroid be cut in the middle of A S by a Plain z z parallel to H T, the Content of the Fruſtum Z A Z, to the Content of a Cylinder whoſe Baſe is z z, and Altitude x A, is as 5 to 9, therefore

therefore the Area of the Base z z Multiplied by $\frac{2}{3}$ of x A gives the Content of the Frustum Z A Z.

3. Suppose the Spheroid cut as before, the lower Frustum Z H g E P P Z is equal to a Cylinder, whose Base is z z, and Altitude x E, (*vid.* Dr. *Wallis*'s Treatise of Algebra, page 312.) these three proportions do likewise hold in a Sphere.

To Inch a Spheroid. Figure VIII.

LET H T the greatest Conjugate Diameter be 20 Inches, and A E the Transverse Diameter or Axis be 40 Inches, to find how many Ale-Gallons this Spheroid will contain upon every Inch, Suppose Lines drawn parallel to H T, through every Inch of A S, these Lines may represent so many Diameters of the Spheroid, and the Squares of these may be found by the proportions above mentioned, for finding the Ordinates (B P) of the Ellipsis, in this Example the Square of the first Diameter at one Inch from H T, will be found to be 399, the Square of the second 396, and the third 391; and by these the Content of the 3 first Inches (which are sufficient for our present purpose) may be thus found.

1. To the Square of the first Diameter, *viz.* 399, add 800 (the double Square of H T) the sum is 1199, this divided by 1077 Quotes 1.1132 the Content of the first Inch in Ale-Gallons.

2. To 396 the Square of the second Diameter, add 800, the sum is 1196, this Multiplied by 2 gives 2392, Divide by 1077 the Quotient will be 2.2209, the Content of the two first Inches.

3. To 391 the Square of the third Diameter add 800, the Sum, *viz.* 1191, Multiplied by 3 is 3573, Divide as above, the Quote

will be 3.3175, the Content of the three firſt Inches. Having got theſe three numbers, take the firſt from the ſecond, the remainder is 1.1077, the Content of the ſecond Inch, in like manner take the ſecond from the third, the remainder will be 1.0966 the Content of the third Inch, So the Content of

the	Firſt	Inch is	1.1132
	Second		1.1077
	Third		1.0966

As in the ſecond Column of the following Table. Take the ſecond Inch from the firſt, the difference .0055, take the third Inch from the ſecond, the difference is .0111, ſet theſe two differences in the third Column: then take .0055 from .0111, the difference is .0056, which place in the fourth Column, this will be the ſame throughout: being thus prepared add .0056 to .0111, the Sum is .0167, take this from the third Inch, *viz.* 1.0966, the remainder is 1.0799, the Content of the fourth Inch, and ſo of the reſt as in the Table.

Inch

Inch.	Content upon every Inc.	First Differ	Sec. Differ
1	1.1132		
2	1.1077	.0055	.0056
3	1.0966	.0111	.0056
4	1.0799	.0167	.0056
5	1.0576	.0223	.0056
6	1.0297	.0279	.0056
7	0.9962	.0335	.0056
8	0.9571	.0391	.0056
9	0.9124	.0447	.0056
10	0.8621	.0503	.0056
11	0.8062	.0559	.0056
12	0.7447	.0615	.0056
13	0.6776	.0671	.0056
14	0.6049	.0727	.0056
15	0.5266	.0783	.0056
16	0.4427	.0839	.0056
17	0.3532	.0895	.0056
18	0.2581	.0951	.0056
19	0.1574	.1007	.0056
20	0.0511	.1063	
	14.8350		

The Numbers in the second Column of this Table shew the Content of every Inch of the half Spheroid H h A t H, the Sum of these is 14.835, this doubled is 29.67 the Content of the whole, which is not a quarter of a Pint less than the Content found by the former Rule, for the Area of the Circle HT is 1.114, this Multiplied by $\frac{2}{3}$ of AE, *viz.* 26.66 gives 29.699.

Note,

If Sb be 14 Inches, we may find the Content of the Fruſtum H h t H, by adding the firſt 14 Numbers in the ſecond Column of the foregoing Table together, the Sum will be 13 0459 Ale-Gallons, and this doubled is 26.0918 the Content of the middle Fruſtum h t T P g, which may repreſent a Cask, as the half of it may repreſent a Tun, and if a Tun be in this form, having for one of its Baſes the greateſt Circle of the Spheroid, the Content may be found as in Problem I. Sect. IX. of the foregoing Tract; and by the VII. Problem of the ſame Section, the Diameters, and conſequently the Content upon every Inch of ſuch Tun may be readily found.

In the Definitions of the Solids here treated of, they are ſaid to be made by a plain figure, being turned about a Right Line given; and theſe Solids do all of them (except the Spheroid) take their denomination from the Figure turned about, as Parabolick Conoids, and Spindles from Parabola's, Hyperbolicks from Hyperbola's, and even of the Spheroid it ſelf if a part be taken towards the Vertex A, it is by ſome called an Elliptical Conoid.

Now if the Area or Content of any plain Figure, and the diſtance of its Center of Gravity from the Line about which it is turned be given, the Content of the Solid deſcribed by the turning about of ſuch plain Figure is alſo given; for,

If a Plain Figure, turned about a Right Line, given in the ſame Plain (which ſhall not cut the Figure given) ſhall deſcribe a Solid Figure: this Solid

Solid is equal to a Prismatick Solid upon the same or an equal Base, having Altitude equal to the Perifery (perfect or imperfect, as the Conversion was perfect or imperfect) which is described by the Center of Gravity of the Figure given to be turned. See this demonstrated by Dr. *Wallis* in his *Mechanica*, page 197. who hath shewn in the same Book how to find the Center of Gravity of any Plain Figure, and proved that the Center of Gravity of a Semi-Parabola (suppose A P o in Fig. IV.) is distant from the Vertex A, $\frac{1}{3}$ of the Altitude, and from the Line A B, $\frac{1}{8}$ of the Latitude, that is, $\frac{1}{8}$ of P o, therefore the Center of Gravity of the Semi-Parabola, will be in the point a; and of the whole Parabola in the point x.

Now if the Parallelogram A N o P, be turned about the Line A P, it will describe the Cylinder N o o V, and by the same Conversion the Semi-Parabola A O P, will describe the Parabolick Conoid A o P o, upon the same Base o o having Altitude A P, the same with the Cylinder, and from the proportion of the Semi-Parabola, to the Parallelogram Circumscribed, and from the Center of Gravity of one to that of the other is proved, that the Parabolick Conoid A o P o, to the Cylinder N o o V is as 1 to 2, that is, the former is half of the latter. Dr. *Wallis's Mechanica*, Part 2. Problem 12.

Now whilst the Figure is turning about, the Center of Gravity a, will describe a Circumference, whose Radius is x a, and the length of this Circumference Multiplied by the Area of the Semi-Parabola A o P, produces the Content of the Parabolick Conoid o A o P.

Moreover,

Moreover if the Parallelogram I K H G, (Fig. V.) be turned about the Line G H, it will describe the Cylinder I K M L, and the Parabola G A H (whose Center of Gravity x, is distant from the Vertex A, $\frac{3}{7}$ of the Altitude A Q) will describe the Parabolick Spindle G A H B, which Parabolick Spindle is to the Cylinder Circumscribed as 8 to 15, as is proved in the place above cited.

Now whilst the Figure is turning about, the Center of Gravity x, will describe a Circumference, whose Radius Q x being given, the length of the Circumference is also given, which Multiplied by the Area of the Parabola G A H, the Product is the Content of the Parabolick Spindle G A H B.

I might now compare these Rules with those I have given above, but I will first give an Example in two Solids more commonly known, *viz.* a Cylinder and a Sphere, now it is well known to the meanest Geometrician, that a Sphere to a Cylinder of the same Base and Altitude is as 2 to 3. Suppose the Diameter and Altitude of a Cylinder be each 12 Inches, the Content will be found to be 1357.1539 Solid Inches; $\frac{2}{3}$ of this is 904.7692 the Content of a Sphere whose Diameter is 12 Inches.

If B D (Fig. 9.) be 12, and A C 6, the Area of the Parallelogram B E F D is 72; the Center of Gravity is in the middle at v, so the Radius A v being 3 the length of the Circumference v n r e (which is described by the point v, whilst the Parallelogram is turning about) is 18:84936, this Multiplied by 72 (the Area of the Parallelogram) gives 1357.1539 the Content of the Cylinder as above.

Again,

Again, the Center of Gravity of a Semi-Circle (according to Dr. *Wallis*) is thus formed,

As the Semi-Periphery is to the Diameter,
So is $\frac{2}{3}$ of the Radius, to the distance of the Center of Gravity from the Center of the Circle.

In this example the Diameter B D is 12, and the Semi-Periphery B C D, 18.8495, therefore

As 18.8495 the Semiphery, to 12 the Diameter,
So is $\frac{2}{3}$ of the Radius, *viz.* 4 to 2.54649 which is equal to A S, the distance of the Center of Gravity from the Center of the Circle. Now whilst the Semi-Circle is turning about, the Center of Gravity S, will describe the Circumference S z x c, whose length is 16 Inches, because the Radius A S is 2.54649.

The Area of the whole Circle is 113.09616, the half is 56.54808 the Area of the Semi-Circle B C D, this Multiplied by 16, gives 904.76948 the Content of the Sphere, which is but $\frac{2\frac{1}{2}}{100000}$ of a Cubick Inch more than the Content found as above.

If this Rule be applied to the other Solids, the like agreement will appear: for instance,

In Fig. IV. P o is 20 Inches, and P A 40, therefore the Parallelogram N o P A is 800; $\frac{2}{3}$ of this is 533.333 the Area of the Semi-Parabola o o A P.

The Center of Gravity a, is distance from A P $\frac{3}{8}$ of P o, therefore x a is 7.5: Now if the Semi-Parabola be turned about the Line A P, the Center of Gravity a, will describe a Circumference whose length will be 47.1248, this Multiplied by the Area of the Semi-

Semi-Parabola, *viz.* 533.333 gives 25132.784, the Content of the Parabolick Conoid, whose Base is o P o, and Altitude P A, and this is half the Circumscribed Cylinder N o o V, for the Diameter o P o is 40 Inches, therefore the Area of the Base is 1256.6384, this Multiplied by P A the Altitude, *viz.* 40, gives 50265.536 the Content of the Cylinder, the half is 25132.768, the difference is but $\frac{16}{1000}$ of a Cubick Inch, and there would be no difference at all if the Work could be performed by whole Numbers. I might further shew the agreement of these Rules, in the Parabolick Spindle, and other Solids, but by what hath been already said, I presume the method is so plain as not to need more Examples.

The END.

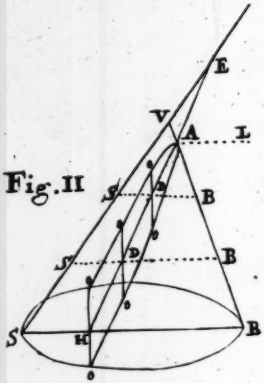


Fig. II

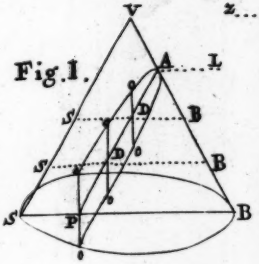
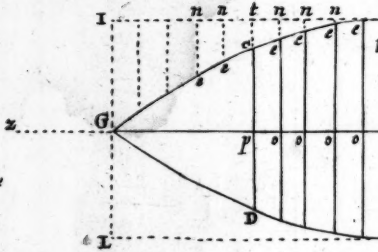


Fig. I.



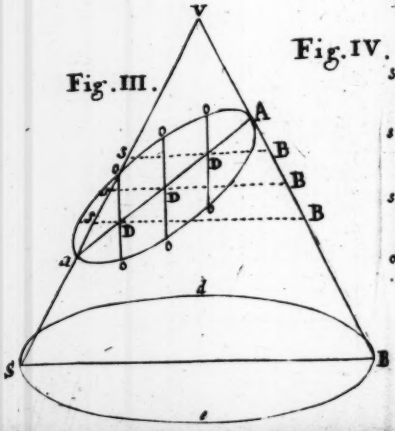


Fig. III.

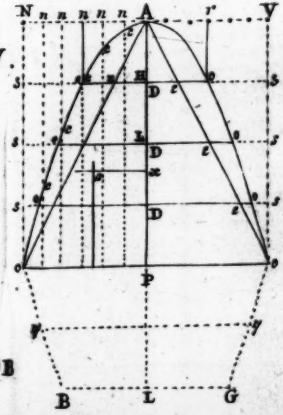


Fig. IV.

Fig. VI.

Place this at th

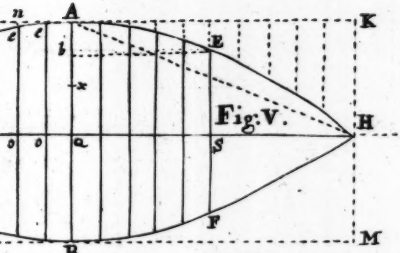


Fig. V.

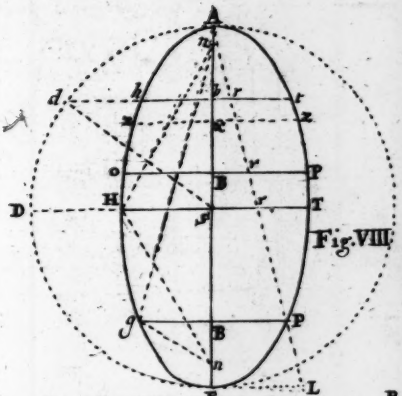


Fig. VIII.

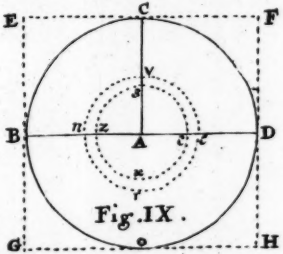


Fig. IX.

I.

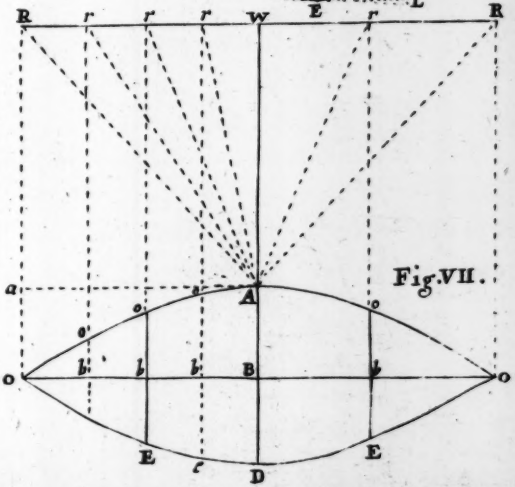


Fig. VII.

at the end of the Appendix.

ERRATA.

Page 12. *Line* 13. for .0775 *Read* .0375. *p.* 44. *l.* last but two, *r* .5. *p.* 49. *l.* 11. *r.* 1002.6. *p.* 55. *l.* 3. *r.* 1.72047. *p.* 54. *l.* last, *r.* .68819. *p.* 57. *l.* last but one, *r.* the Product. *p.* 76. *l.* last but two, for ſame, *r.* ſum. *p.* 86. *l.* 4. for D. *r.* b. *p.* 94. *l.* 12. *r.* 91. *l.* 14. for 91. *r.* 93. *p.* 98. *l.* 7. *r.* Area's. *p.* 99. firſt Col. of the Table, for 9. *r.* 6. *p.* 102. *l.* 9. for is *r.* if. *p.* 103. *l.* 29. dele to. *p.* 106. *l.* 23. after Sum, *r.* and half Sum. *p.* 113. *l.* 5. for length, *r.* tenths. *p.* 115. *l.* 16. *r.* a Spheroid. *p.* 126. *l.* 5. *r.* Squares. *p.* 132. *l.* 16. for 144. *r.* 156.

In the Table of Area's.

Diam.	Depth.	Read thus.
13.5	1	0.05076
13.4	9	4.43
21.7	3	3.93
25.9	7	13.08
26.8	1	2.0003
26.5	1	2.0153
32.9	1	3.0146
ibid.	3	9.04
35.8	7	24.99
49.4	9	61.17
55.8	9	78.04
55.9	9	78.33
56.4	6	53.16
56.9	2	18.03
59.4	1	9.8268
61.0	9	93.27
61.3	9	94.19
62.9	2	22.04
68.9	9	118.59
70.5	8	110.74
71.9	5	71.99
72.9	9	133.21
75.7	6	95.76
76.6	4	65.33
77.9	1	16.9011

Diam.	Depth.	Read thus.
83.2	3	57.84
88.0	9	194.11
88.1	9	194.55
88.2	9	194.99
88.3	9	195.44
88.4	9	195.88
90.9	9	207.12
92.7	9	215.40
93.9	9	221.01
101.0	2	56.82
102.0	9	260.78
104.0	8	240.98
113.4	5	179.08
114.5	6	219.08
114.6	8	292.62
118.6	5	195.88
119.9	9	360.34
121.6	8	329.46
123.4	1	42.4103
123.9	7	299.29
124.6	7	302.67
125.4	9	394.16
142.4	7	395.33
148.4	1	61.3350